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CLAVIS MATHEMATICA

denuò LIMATA,

Sive potius

FABRICATA.

Cui accedit

Tractatus de Reſolutione Æquationum qualitercunque adfectarum in numeris:

Bib: Sec. Coll. Bon. Soc. J.

Et

Declaratio tum decimi elementi *Euclidis* de lateribus incommenſurabilibus: tum decimi tertii & decimi quarti elementi de quinque ſolidis Regularibus.

Atque hic paſſim

Logiſticæ decimalis, & *Logarithmorum* Doctrina intexitur.

Autore Gulielmo Oughtredo *Anglo.*

LONDINI,

Excudebat *Thomas Harper*, ſumptibus *Thomæ Whitakeri*, apud quem venales ſunt in Cœmiterio *D. Pauli.* 1648.

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CLAVIS MATHEMATICÆ DENUO LIMATA.

Cap. I. *De Notatione.*

1. TAbella admodum utilis, non modò pro numerorum Notatione, quam primâ faciè exhibet; ſed etiam in omni computatione per numeros tum communes, tum figuratos, tum artificiales, qui vulgò Logarithmi dicuntur.

Integri — Partes.

9	8	7	6	5	4	3	2	1	0	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	$\bar{8}$	$\bar{9}$	&c.
M	M	M	M	M	M	M	C	X	I	X	C	M	M	M	M	M	M	M	
M	M	M	M	C	X	I							X	C	M	M	M	M	
M	C	X	I													X	C	M	

2. In hâc tabellâ numeri ſuperiores ſunt Indices ſive exponentes terminorum utrinque ab unitate continuè proportionalium, affirmativi in integris, negativi in partibus. Eſtque progreſſio in decuplâ ratione verſus ſiniſtram, & in ſubdecuplâ verſus dextram; ſicut literæ numerales ſubſcriptæ oſtendunt. Eſt igitur pro-

greſſio

greſſio ab unitate in integris, 1, 10, 100, 1000, 10000: Et in partibus, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$: Et ſic in infinitum.

3. Atque hoc modo in omni aliâ Progreſſione, terminis ab unitate quacunque ratione ſive creſcentibus, ſive decreſcentibus, Indices ſui erunt adponendi.

4. Tabellam quidem in decimali ratione ordinavi, tum ut numerorum quorumcunque (ſive Integri ſint, ſive partes, ſive mixti) valores per gradus & periodos æſtimentur: tum quia Logiſtica hæc decimalis ſexagenaria, in computationibus Aſtronomicis multò facilior eſt atque concinnior. Hoc planè perſpexit, quicunque is fuit, qui primus canonem Sinuum à ſemidiametro 60, ad I cum circulis annexis, revocavit. Utinam idem etiam in aliis canonibus fierit.

5. Partes decimales ſcribuntur in unâ lineâ cum integris, diſtinguuntur autem lineolâ rectangulari, quæ idcircò ſeparatrix dicitur. Et quemadmodum in integris, quilibet ab unitatum loco gradus augetur versùs ſiniſtram decuplando: ſic in partibus decimalibus, quilibet ab unitatum loco gradus minuitur verſus dextram ſubdecuplando.

6. Partes decimales denominationem ſuam ſortiuntur à loco figuræ ſuæ ultimæ: ut 0|5 ſunt 5 decimæ partes: 0|56 ſunt 56 centeſimæ partes: 0|056 ſunt 56 mileſimæ partes, & ſic de reliquis omnibus.

7. Circuli ante integros, vel poſt partes decimales nihil valent: at verò poſt integros, & ante partes decimales (hoc eſt utrinque lineæ ſeparatrici proximi) vim

ſuam

ſuam retinent : nam gradus conſtituunt quibus figurarum valores cenſentur:ut, 0005,ſignificant tantummodo 5 : & 0|500, 5 decimæ partes.

8. Quare in partibus decimalibus ſcribendis,linea ſeparatrix ſemper adponatur ; & loci, ſi qui ſunt, vacui, circulis ſuppleantur : ut 0|00005 ſunt 5 centies milleſimæ partes.

9. Signum addendi ſive adfirmationis eſt + plus, ſive pl: ut 34, vel + 34.

10. Signum minuendi ſive negationis eſt — minus, ſive mi : ut—34, negantur omninò eſſe.

11. Pertinet autem ſignum ad magnitudinem ſequentem, cui præfigitur. Et omnis magnitudo, cui non eſt præfixum ſignum negationis, intelligitur eſſe adfirmata,& habere ſignum +, licet non ſit expreſſum.

12. Et nota quod ſignis + & − utor, quando ſimplex magnitudo adfirmatur vel negatur de ſimplice: ſignis autem pl : & mi, quando magnitudo compoſita adfirmatur vel negatur de ſimplice, vel ſimplex de compoſitâ.

13. Magnitudines denotari poſſunt vel numeris menſuram ipſarum ſignificantibus, vel etiam ſpeciebus : ut linea longa ſeptem uncias, deſignatur vel per 7; vel per unam aliquam literam aut notam, A, B, C, &c; vel per duas literas terminis lineæ adſcriptas,AB, BC, CD, &c pro libitu : modò memoriâ teneas pro quâ magnitudine ſpecies quælibet ſtatuitur.

14. Specioſa hæc Arithmetica arti Analyticæ(perquam ex ſumptione quæſiti, tanquam noti, in veſtiga-

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tur quæſitum) multò accommodatior eſt, quam illa numeroſa. Nam in numeroſâ, numeri à novo, quem proferunt, ita abſorbentur, ut penitùs diſpareant, nec ullum ſui veſtigium relinquant: At in ſpecioſâ, permanent ſpecies ſine aliquâ mutatione, ſpecimen exhibentes totius operationis: unde non ſolùm in quæſiti notitiam ducunt, ſed etiam Theorema generale pro ſolutione conſimilium quæſtionum, in aliis magnitudinibus datis, edocent.

Cap. II. *De Additione.*

1. Numerus inventus per Additionem, dicitur Summa, vel Aggregatum. Ut 3 & 7 conſtituunt 10.

2. Additio incipit ad dextram, & ſummas ſingulorum locorum particulares inventas ſubſcribit, in locis ſuis propriis.

3. In Additione omnes numeri dati ſimul æquantur Summæ.

Exempla Additionis.

79403	3794,236	l.	s.	d.
8956	584,3	17	13	4
67293	947,08	9	16	7
5087		238	09	6
160739	4720,7439	70	00	10
	48,5	48	10	3
	10094,8599	384.	10	6

4. Additio

4. Additio ſpecioſa conjungit omnes magnitudines datas ſervatis ſignis

ad	3A	A	5A	3A	A
adde	A	−A	−3A	−5A	E
Summa	3A+A	A−A	5A−3A	3A−5A	A+E
hoc eſt	4A	0	2A	−2A	

ad	A+B	A+B	Sic in Indicum Additione: 3 | 3, 2 | 2, 1 | 1
adde	A−B	A−C	
Summa	2A	2A+B−C	

Cap. III. *De Subductione.*

1. Numerus inventus per Subductionem dicitur Reliquus, vel Differentia, vel Exceſſus. Ut è 7 tolle 3, reſtat 4.

2. Subductio incipit ad dextram, & differentias ſingulorum locorum particulares inventas ſubſcribit, in locis ſuis propriis.

3. In Subductione, numerus ſubducendus, unà cum differentia, æquatur numero ex quo.

Exempla Subductionis.

		l.	s.	d.
347206836	3794,236	17	13	4
6807592	947,08	9	16	7
340399244	2847,156	7	16	9

 4. Sub-

4. Subductio ſpecioſa conjungit utramque magnitudinem datam, mutatis omnibus ſignis magnitudinis ſubducendæ.

Ex- 4 A	3 A	5 A	A
tolle A	5 A	−3 A	E
Reſtat 4 A−A	3 A−5 A	5 A+3 A	A−B
hoc eſt 3 A	−2 A	8 A	

Ex- A	A	Sic in Indicum ſubductione.	3 | 3
tolle B+C	B−C		2 | 2
Reſtat A−B−C	A−B+C		5 | 5

Cap. IV. *De Multiplicatione.*

1. Numerus inventus per Multiplicationem, dicitur Factus, vel Productus, vel Rectangulum, vel Planum. Nam unus è numeris propoſitis habetur pro longitudine, alter pro latitudine: & numeri propoſiti dicuntur Factores atque Latera. Maxima quippe binarum magnitudinum poteſtas, eſt figura ex ipſis compoſita, cujus anguli ſunt recti, & latera parallela.

2. Multiplicatio incipit ad dextram, & ſingulas figuras unius numeri dati, in ſingulas alterius figuras ducit. & factos demum, habitâ locorum ratione, in unam ſummam colligit. Et ſi partes decimales numeris propoſitis ſint admixtæ, è toto facto tot locos lineâ ſeparatrice abſcindit, quot ſunt loci partium in utroque

utroque factore. Nam in Multiplicatione Index cujuſque particularis figuræ facti, invenitur addendo Indices figurarum multiplicatæ & multiplicantis, ſic 58,73 ductus in 600, facit 35238. Nam Index figuræ $\bar{6}$ in 600, eſt 2: & Index ultimæ figuræ 3 in 58,73 eſt $\bar{2}$, addantur Indices 2 & $\bar{2}$, extabit 0 pro Indice ultimæ figuræ facti 35238: quæ idcirco pertinet ad locum unitatum. Et conſimilis reliquarum figurarum in facto cenſura gradualis inſtitui poterit.

3. Si è numeris propoſitis, unus, vel uterque, adjunctos habeat ad dextram circulos: omiſſis circulis, fiat ipſorum numerorum Multiplicatio: & facto demùm tot inſuper integrorum loci accenſeantur, quot ſunt omiſſi circuli in utroque factore.

4. In Multiplicatione eſt, ut unitas, ad unum è factoribus: Sic alter è factoribus, ad factum. Ut ſi ducatur 4 in 6 fiet 24: Eſt igitur 1.4 :: 6.24: vel 1.6 :: 4.24.

Exempla Multiplicationis.

4576	580,34
892	47,5
9152	290170
41184	406238
36608	232136
4081792	27566,150

358	58,73
600	600
214800	35238,0

5. Contractio Multiplicationis, in Logiſticâ valdè utilis, ſic eſt. Si inſtituto tuo ſufficiat habere factum non integrum, ſed multatum aliquot ex ultimis figuris: ſtatues unitatis locum minoris numeri, ſub illâ figurâ majoris, cujus Index æqualis ſit numero figurarum, vel abſcindendarum in integris vel relinquendarum in partibus decimalibus: Et reliquas figuras minoris numeri, ſub numero majore ordine inde contrario. Tum in multiplicando incipies ubique ad illam figuram majoris numeri, quæ eſt ſupra eam figuram minoris, quâ multiplicatur: habitâ tamen ratione incrementi, quod ex ſubſequentibus figuris majoris numeri ſuppeditatur. Hujus compendii caſus ſunt quatuor.

Caſus I. Si velis factum habere purum à partibus: Statues unitatis locum minoris ſub unitatis loco majoris. Ut in exemplo, ubi 246|914 ductus in 35|27 producit 8708 integros, abſciſſis omnibus partibus decimalibus.

246|914
72|53
7407
1235
49
17
8708

Caſus II. Si velis habere factum cùm locis aliquot partium puta quatuor: Statues unitatis locum minoris numeri ſub quarto loco partium majoris. Ut in priore exemplo, factus erit 8708,6568 mixtus cum quatuor locis partium.

246,914
72,53
74074200
12345700
493828
172840
8708,6568

Caſus III. Si velis factum multatum aliquot locis integrorum, puta quinque: ſtatues unitatis locum minoris numeri loco quinto ante unitatis locum majoris. Ut in exemplo, ubi 80902 ſinus graduum 54 multiplicandus eſt per 39875 ſinum maximæ declinationis 23° 30: prodibit 32266 ſinus declinationis ſolis ad ♉ 24°.

80902
57893
24271
7281
647
63
4
32266

Caſus IV. Si velis factum multatum locis integrorum, puta quinque, reparari aliquot locis partium, puta quatuor. Quia 5 + 4 = 1: Statues unitatis locum minoris numeri uno loco ante unitatis locum majoris. Ut in exemplo ubi ſinus 42262 multiplicatur per 0,0064, ita ut abſciſſis à facto quinque figuris ultimis, reſtituantur quatuor loci partium: Factus erit 0,0027.

42262
46000
25
2
0,0027

6. Multi-

6. Multiplicatio *ſpecioſa* connectit utramque magnitudinem propoſitam cum notâ in vel ×: vel plerumque abſque notâ, ſi magnitudines denotentur unicâ literâ. Et ſi ſigna ſint ſimilia, producta magnitudo erit adfirmata: ſin diverſa, negata. Effertur autem per in.

Et nota, quòd A in A, ſive A × A, ſive A A, eſt Aq. AAA ſive AqA, eſt Ac. AAAA, ſive Aq Aq, ſive AcA, eſt Aqq. AAAAA, ſive AcAq, ſive AqqA, eſt Aqc, AAAAAA, ſive Ac Ac, ſive Aqq Aq, ſive AqcA, eſt Acc, &c. Nam poteſtas quælibet ſuperior fit ex duabus inferioribus, quarum dimenſiones ſimul æquantur numero dimenſionum ſuperioris. Quot autem magnitudines ſunt quæ multiplicantur, totidem ſunt dimenſiones.

Duc. A	A+E	A−E	A+E+I	B+1
in E	B	B	Z	A
fiet AE	BA+BE	BA−BE	ZA+ZE+ZI	BA+A

Duc. 3A	AE	AE	A+E	A+E
in 2A	A	AE	A+E	A−E
fiet 6Aq	AqE	AqEq	Aq+AE	Aq+AE
			+AE+Eq	−AE
				−Eq
			Aq+2AE+Eq	Aq−Eq

Ad hunc etiam modum Multiplicatio fiet, ſi magnitudines conſtent binis literis. Ut ſi latus

AB

A B + C D multiplicandum ſit in ſe, producetur quadratum ABq + 2 AB × CD + CDq.

Cap. V. *De Diviſione.*

1. Numerus inventus per Diviſionem dicitur quotus, vel etiam Parabola: quia oritur ex adplicatione numeri plani ad longitudinem datam, ut inveniatur latitudo congrua. Et ſi numerus ad numerum adplicetur cum lineolâ interjectâ, oſtendit quod numerus ille ſuperior dividendus ſit per inferiorem, ad quem adplicatur: ut $\frac{8}{4}$ & $\frac{5}{12}$.

2. Diviſio incipit ad ſiniſtram: & poſtquam ex dividendo ſufficientem diviſori dividuum diſtinxerit, & ſub ipſo diviſorem ſubſcripſerit, vel ſaltem ſubſcriptum cogitaverit: ſingulas figuras diviſoris ex ſingulis ipſius dividui figuris ſupra ſtantibus, æqualiter, quoties fieri poterit, tollit: Tum diviſore per quotum inventum multiplicato, factoque ablato ex dividuo, diviſorem in locum proximè ſequentem promovet, novamque uti prius diviſionem inſtituit; donec totum dividendum percurrerit. Quilibet autem quotus particularis inventus, ejuſdem debet eſſe loci, ſive gradus, cujus eſt figura dividendi, quæ ſtat, vel cogitatur ſtare ſupra unitatis locum diviſoris. Nam in Diviſione, Index cujuſque particularis figuræ Quoti, invenitur tollendo Indicem figuræ dividentis ex Indice figuræ diviſæ. Sic 171|4 diviſus per 857, dat 0,|2 pro Quoto. Index enim primæ figuræ dividuæ 17 eſt 1; & Index

primæ

primæ figuræ diviſoris 8 eſt 2: Tollatur 2 ex 1, reſtabit 1 pro Indice primæ figuræ: quæ idcircò pertinet ad locum primum partium decimalium.

3. Et ſi diviſor adjunctos ſibi habeat ad dextram circulos; omiſſis circulis, & abſciſſis totidem ultimis figuris dividendi, in numeris reliquis fiat diviſio. In fine autem diviſionis reſtituendi ſunt, tum omiſſi circuli, tum figuræ abſciſſæ.

4. In Diviſione eſt, ut Diviſor ad unitatem, ſic dividuus ad Quotum: vel ut dividuus ad diviſorem, ſic Quotus ad unitatem. Ut diviſo 24 per 6, quotus erit 4: Eſt igitur 6.1::24.4: Item 24.6::4.1.

5. Si magnitudo facta ſit ex duabus magnitudinibus, una ex iis ipſam per alteram metietur.

6. In Multiplicatione, atque Diviſione, unitas nihil mutat.

7. Si numerus numerum multiplicet, idemque factum dividat, nihil fit. Nam quod Multiplicatio conficit, Diviſio diſſolvit. Quare in adplicatione magnitudinis ad magnitudinem, ſi eadem magnitudo ſit tum ſupra lineam, tum infra, expungatur utrobique.

Exempla

Exempla Diviſionis.

12
8921317
297) 187135075 (630084 $\frac{117}{297}$
178213768
892118
2901
435257
580,34) 27566,150 (47,5
23213680
406237
2901

187135075 (630084 $\frac{117}{297}$
297
1782 6,000)4320|765
893 720|1275
297
891
2507
297
2376
1315
297
1188
127

8. Ali-

8. Aliquando numerus aliquis dividi poſtulatur per numerum irrationalem, vel infinitum, ſive integer ſit, ſive mixtus. Atque in hoc caſu, ſumptis, quot opus eſt; è primoribus figuris diviſoris pro primo diviſore, per ipſas divides numerum propoſitum : deinde pro ſingulis particularibus diviſionibus ſubſequentibus, diviſorem minues amputando verſus ſiniſtram totidem ultimas figuras, donec quotum ſatis amplum inveneris : ut ſi dividantur 467023 per numerum infinitum 357,0926425 Quotus erit 1307,80 ferè.

```
                      17
                     303
                    2803
                  209930
357,0926425)467023(1307,80—
                  357093
                  107127
                    2500
                     286
```

Pulcherrima hæc eſt Diviſionis contractio, & maximi uſus in computationibus Aſtronomicis. Ut ſi per 137638 dividendus ſit 126223 ductus in ſinum totum, hoc eſt auctum quinque circulis : Appones tantummodò unum circulum : & pro quatuor reliquis minues diviſorem. Ut

137638) 1262230 (91727.

9. Divi-

9. Diviſio *ſpecioſa* ſtatuit magnitudinem dividentem ſub dividendâ, cum lineolâ interjectâ: tùm conſiderat an magnitudo aliqua utramque communitèr multiplicaverit; atque ipſam utrobique expungit. Diviſio etiam in iiſdem dat +, in diverſis −: effertur autem per ad.

Adplica	AE	BAc	BA+A	BA−CA	6Aq	ſcil. $\frac{2 \times 3 Aq}{3 A}$
ad	A	Aq	A	B−C	3A	
Oritur	E	BA	B+1	A	2A	2A

Cap VI. *De Proportione.*

1. Si è quatuor numeris datis, primus ita ſe habeat ad ſecundum, ut tertius ad quartum: dicuntur quatuor illi numeri eſſe proportionales. Numerorum autem ad ſe invicem habitudo invenitur dividendo antecedentem per conſequentem: ut 31 ad 7 ratio eſt $4\frac{3}{7}$, hoc eſt quadrupla ſupertripartiens ſeptimas.

2. Quare ſi numerus duos numeros multiplicet, facti erunt multiplicatis proportionales. Et ſi numerus duos numeros dividat, quoti erunt diviſis proportionales.

Ut $4 \times \left\{ \begin{matrix} 7. \\ 9. \end{matrix} \right. \begin{matrix} 28 \\ 36 \end{matrix}$ & $4) \begin{matrix} 28(7. \\ 36(9. \end{matrix}$ Item $A \times \left\{ \begin{matrix} B.\ BA. \\ C.\ CA. \end{matrix} \right.$ &

$A) \begin{matrix} BA.\ B \\ CA.\ C. \end{matrix}$

3. Quare

3. Quare ſi quatuor numeri ſint proportionales, factus ab extremis æquatur facto à mediis. 7.9::7×4. 9×4::28.36. At 7×9×4 = 9×7×4.

4. Hinc ſequitur aurea (quæ dicitur) regula Proportionis. Si è tribus numeris datis, rectangulum ſub ſecundo & tertio adplicetur ad primum : hoc eſt, ſi ſecundus multiplicet tertium, & primus dividat factum : quotus erit tribus datis quartus proportionalis. Tres numeri dati ſunto 7, 9, 28: & pro quarto quæſito ſtatuatur Q. Eſt igitur 7.9::28.Q. Quare 7Q = 9×28. Ideoque $\frac{9\times28}{7}$ = Q. Item 5.12::8. $\frac{8\times12}{5}$, hoc eſt $19\frac{1}{5}$.

5. E tribus numeris datis ad quartum Proportionalem inveniendum, duo primi innuunt rationem, & reliquus ingreditur quæſtionem, eſtque in Proportione Directâ primus terminus (ſive Diviſor) homogeneus ei per quem fit quæſtio : At in Proportione Reciprocâ primus terminus (ſive Diviſor) ipſe eſt per quem fit quæſtio.

6. Directa quidem Proportio eſt, quando terminus is per quem fit quæſtio, quô major eſt, eô quartum majorem requirit: & quo minor eô minorem.

7. Reciproca Proportio eſt, quando terminus is per quem fit quæſtio, quô major eſt, eô quartum minorem requirit: & quô minor, eô majorem.

8. Proportio continua ∺ eſt, quando termini omnes medii inter primum & ultimum, rationum ſunt tum conſequentes, tum antecedentes. Ut 8, 12, 18, 27, ſunt ∺. Nam 8.12::12.18::18.27.

Item

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Item $\alpha, \beta, \frac{\beta q}{\alpha}, \frac{\beta c}{\alpha q}, \frac{\beta qq}{\alpha c}, \frac{\beta qc}{\alpha qq}$ &c. ſunt $\div\!\div$

Quare ſi in hâc ſerie ultimus terminus ſit ω, & ſumma omnium terminorum totius progreſſionis ſit Z: erit $Z - \omega$ ſumma omnium antecedentium : & $Z - \alpha$ ſumma omnium conſequentium.

9. Si quatuor magnitudines ſint proportionales, $A . \alpha :: B . \beta$: etiam alternè, & inverſè, & compoſitè & diviſim, & conversè & mixtim proportionales erunt,

alternè, $A . B :: \alpha . \beta$.
inverſè, $\alpha . A :: \beta . B$.
compoſitè, $A + \alpha . \alpha :: B + \beta . \beta$.
vel, $A + B . B :: \alpha + \beta . \beta$.
diviſim., $A - \alpha . \alpha :: B - \beta . \beta$.
vel, $A - B . B :: \alpha - \beta . \beta$.
converſè, $A . A \pm \alpha :: B . B \pm \beta$.
vel, $A . A \mp B :: \alpha . \alpha \mp \beta$.
mixtim, $A + \alpha . A - \alpha :: B + \beta . B - \beta$.
vel, $A + B . A - B :: \alpha + \beta . \alpha - \beta$.

10. Si quotlibet magnitudines ſint proportionales, erit ut unus antecedens, ad ſuum conſequentem; ſic ſumma antecedentium, ad ſummam conſequentium. Eſto $A . \alpha :: B \beta :: C . \gamma :: D . \delta$: erit $A . \alpha :: A + B + C + D . \alpha + \beta + \gamma + \delta$.

Nam $\begin{cases} A . \alpha :: B . \beta . \text{ \& compoſitè} \\ A + B . \alpha + \beta :: (B . \beta)\, C . \gamma . \text{ \&} \\ A + B + C . \alpha + \beta + \gamma :: (C . \gamma)\, D . \delta . \text{ \&c.} \end{cases}$

Item in $\div\!\div$, $\alpha . \beta :: Z - \omega . Z - \alpha$. Quare $\alpha Z - \alpha q = \beta Z - \beta \omega$, vel $\beta Z - \alpha Z = \beta \omega - \alpha q$. Hinc obitèr liquet in-

 ventio

ventio ſummæ omnium terminorum ∹, ſive Progreſſionis Geometricæ : per hanc Regulam $\left\{\frac{\beta a-\alpha q}{\beta-\alpha}\right.=Z$.

11. Si plurium proportionum antecedentes ſint æquales; erit ut unus antecedens, ad ſummam ſuorum conſequentium : Sic alter antecedens ad ſummam ſuorum. Eſto A. B : : $\alpha.\beta$: & A.C : : $\alpha.\gamma$: & A.D : : $\alpha.\delta$: erit A. B + C + D : : $\alpha.\beta+\gamma+\delta$. Liquet ex priore demonſtratione, terminis alternè poſitis.

12. Si binarum rationum conſequentes ſint æquales, ſunt ut antecedentes. Si verò antecedentes ſint æquales, ſunt reciprocè ut conſequentes.

$\frac{7}{1}.\frac{9}{1}::7.9.$ Et $\frac{1}{9}.\frac{1}{7}::7.9.$

13. Si bis quatuor magnitudines ſint ſimiliter proportionales; ipſarum etiam tum ſummo, tum differentiæ proportionales erunt.

14. Si quatuor magnitudines proportionales, per alias quatuor magnitudines proportionales multiplicentur, vel dividantur : etiam Factæ, vel Quotæ, proportionales erunt.

15. Ratio antecedentis ad conſequentem componitur, vel ex ratione antecedentis ad tertium, & tertii ad conſequentem : vel ex ratione tertii ad conſequentem & antecedentis ad tertium. Ut

$7.9::\times\left\{\begin{matrix}7.A.\\A.9.\end{matrix}\right.$ Item $7.9::\times\left\{\begin{matrix}A.9.\\7.A.\end{matrix}\right.$

16. Inventio quarti proportionalis in computationibus Aſtronomicis.

Si

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Si 100000 ſit primus terminus, invenitur quartus per 5, Cap:4. Caſ.3. Ut 100000. 80902 :: 39875. 32260.

Si 100000 ſit ſecundus terminus, invenitur quartus per 8, Cap:5. Ut 137658. 100000 :: 126225. 91706.

17. Inventio partis proportionalis ex datâ differentiâ duorum numerorum in Canone Proſthaphæreſeωr.

In tabulis Prutenicis, Ad epicycli primi Lunæ Anomaliam Gr:62, Proſtph: ablativa eſt Gr: 4,1786: & Differentia ibidem Gr: 0,0433: Quanta ejus pars debetur Anamaliæ Gr: 62,5667. Dic 1. 0,0433 :: 0,5667. 0,0245: per cap.4. ſect.5. Caſ. II. Tum 4,1786 + 0,0245 = 4,2031: quæ eſt Proſthaph: correcta.

Et contrà ſi quæratur Anomalia primi Epicycli Lunæ, congruens Proſthaphæreſi Gr: 4,2031. Proximè minor in Canone eſt Gr: 4,1786, reſpondens Anomaliæ Gr: 62: Eſtque Differentia ibidem Gr: 0,0433. Eſt autem 4,2031 - 4,1786 = 0,0245. Dic igitur 0,0433. 0,0245 :: 1. 0,566 +, partes adjungendæ Gr: 62. Eritque Anomalia quæſita Gr: 62,566 +.

18. Converſio partium Sexageſimarum in Decimales & contra Decimalium in Sexageſimales.

Partes Sexageſimæ, puta 45, convertuntur in Decimales, dividendo per 60. Et contra partes Decima-

les, puta 0,75, convertuntur in Sexageſimas, multiplicando per 60.

Ut 6,0 . 4|5 : : 1. 0,75. } Nam
& 1 . 075 : : 60. 45. }

Diviſio per 60 removet lineam ſeparatricem uno loco verſus ſiniſtram: Et Multiplicatio per 60 promovet lineam ſeparatricem uno loco verſus dextram. Quæ regula notatu digna eſt.

Si verò plures ſint ſpecies Sexageſimales annexæ Integris, puta 127° 32′ 00″ 09‴ 45^iv: hoc uteris compendio. Sub Integris 127 ſtatue ſpecies Sexageſimales deſcenſu obliquo: Tum facto initio ad infimam, ſingulas divide continuè per 6: Et quotus ſuprascriptos ordini proximo ſuperiori adjunges, donec ad Integros perveneris.

127,5333784722
′32,0027083333
″00,1625
6) ‴09,75 * 6
^iv 45

Et contra, ſi partes Decimales dentur, puta 127,5333784729: multiplicabis ipſas continuè per 6; & factos ſubtus ſcribes, amputato in ſingulis ordinibus uno loco verſus dextram; ut deſcenſus obliquus compleatur. Intuere diligenter exemplum.

Gradus Æquinoctialis, cum partibus Decimalibus, puta Grad: 236,4276, convertuntur in partes Decimales diei; dividendo per 360, hoc eſt 6+60.

Et

Et contra, partes, Decimales Diei puta 0,6567433 : convertuntur in Gradus, multiplicando per 360. hoc eſt 60×6. intuere diligenter exemplum.

6) 236,4276
60) 39,4046×6
0,6567433:×60

Gradus Æquinoctialis, cum partibus Decimalibus puta Grad : 236,4276 convertuntur in Horas dividendo per 15, hoc eſt, 3×5.

3) 236,4276
5) 788092×3
15,76184×5

Et contra, Horæ cum partibus Decimalibus, puta 15,76184 convertuntur in Gradus, multiplicando per 15, hoc eſt, 5×3.

• Horæ cum partibus decimalibus, puta Ho: 15,76184 convertuntur in partes Decimales Diei, dividendo per 24, hoc eſt, 4×6.

4) 15,76184

Et contrà partes Decimales Diei, puta 0,6567433+, convertuntur in Horas, multiplicando per 24, hoc eſt, 6×4.

6) 394046×4
0,656433:×6

Summa collecta, puta 191374, convertitur in expanſam, dividendo continuè per 60, & contrà ſumma eadem expanſa, 53 09 34, convertitur in collectam multiplicando continuè per 60.

BIBLIOTECA NAZ. ROMA VITTORIO EMANUELE

Notandum autem hic eſt, quod ſi ſumma collecta ſit unitatum, ſcil: 191374°; expanſa erit 53'' 09' 34°, hoc eſt 53 Sexagenæ ſecundæ, 9 Sexag: 1ª, & 34 unitates. Si verò ſumma collecta ſit ſexageſimarum ſecundarum, ſcil: 191374''; expanſa erit 53° 09' 34''.

	19137'4
60)	318'9|3×60
	53'0

19. Illa quidem proportio rationum fuit æqualitas & dicitur Geometrica, eſt autem alia proportio Arithmetica, quæ eſt æqualitas differentiarum: nempe quando in quatuor terminis, eadem eſt differentia tertii & quarti, quæ eſt primi & ſecundi. Ut 7.4:12.9 vel 7.7−3 : 12.12−3. Arithmeticè proportionales ſunt.

20. Quare è quatuor numeris Arithmeticè proportionalibus, ſumma extremorum æquatur ſummæ mediorum 7 + 12−3 = 7−3 + 12.

21. Et ſi è tribus numeris datis ſecundus addatur tertio, & primus tollatur è ſumma: reliquus erit quartus Arithmeticè proportionalis. Ut ſi dentur 7, 4, & 12: erit 12 + 4 − 7 = 9, qui quartus eſt quæſitus.

22. Eſt etiam proportio Arithmetica continua, ſive Progreſſio, quando omnes termini à primo eâdem continuò exurgunt differentia: Ut 4, 7, 10, 13, 16, 19, &c. Differentia communis omnium eſt 3. Nam in hâc ſerie, primus (& quaſi radix) eſt 4: ſecundus conſtat ex primo & differentiâ unâ: Tertius conſtat

ex

ex primo & differentiis duabus : Et generaliter quilibet terminus conſtat ex primo & ex ſummâ differentiarum, quarum numerus uno minor eſt quam numerus terminorum : Exempli gratia, terminus decimus tertius conflabitur ex primo & differentiis duodecim, quarum ſumma eſt 36. Eſt igitur 4+36, hoc eſt 40, terminus decimus tertius.

23. Si in Progreſſione Arithmetica, primus terminus addatur ultimo, & ſumma ducatur in numerum terminorum : factus erit duplicata ſumma totius Progreſſionis : Nempe 40 + 4 in 13=572, quæ ſummæ eſt terminorum duplicata.

24. Si ſupra ſeriem terminorum in Progreſſione Geometrica, ſtatuatur pro Indicibus, ſeries terminorum qualiumcunque Progreſſionis Arithmeticæ : quibuslibet quatuor numeris in Arithmetica proportione reſpondebunt quatuor numeri Geometricè proportionales.

Indices, *6.* 8. 10. 12. 14. *16.* 18. 20.
Termini, 5. 15. 45. 135. 405. 1215. *3645*. 10935

Quia $10 + 16 - 6 = 20$; Erit $\frac{45 \times 1215}{5} = 10935$.

Atque hinc patet inventio termini cujuſvis in Progreſſione Geometrica.

25. Eſt etiam tertia Proportio, Muſica dicta, Quando in quatuor numeris, eſt ut Primus ad Quartum: ſic differentia primi & ſecundi, ad differentiam Tertii & Quarti. Ut 5,8,12,30, ſunt muſicè proportionales: quia 5. 30 :: 8-5. 30-12 :: 3. 18. Item in ſpecie-

 bus

bus A, M, N, E; Eſto A. E :: M — A. E — N. Quare AE—AN=ME-AE. Terminis hiſce ritè ordinatis Regula erit, $\frac{AN}{2A-M}=E$. & $\frac{EM}{2E-N}=A$.

In verbis ſic, ſi rectangulum ſub primo & tertio dividatur per exceſſum primi duplicati ſupra ſecundum: quotus erit quartus in Muſica proportione. Quare oportet terminos ſic dari, ut primus duplicatus excedat ſecundum.

Cap. VII.

De Maxima Communi Menſura: quâ numeri dati reducuntur ad minimos terminos ejuſdem rationis.

1 Maxima duorum numerorum communis menſura invenitur perpetua diviſione majoris per minorem, & diviſoris per reliquum. Nam diviſor ille qui primus dividuum ſuum metitur, abſque ullo reliquo, maxima erit utriuſque numeri dati communis menſura. Ut numerorum 899 & 744 maxima menſura invenietur 31.

```
       31  124  155
31) 124)155)744)899(1414
    124  124  620  744
```

2 Nu-

2 Numerorum reductio ad minimos terminos ejuſdem rationis fit dividendo utrumque per maximam ipſorum communem menſuram, ut 899 & 744 reducuntur ad 29 & 24, qui minimi ſunt termini in eadem ratione, diviſo utroque per 31 maximam utriuſque menſuram. Sic $\frac{3Aq}{6A}$ reducuntur ad $\frac{A}{2}$ dividendo utrumque terminum per 3A. Et $\frac{4Acc}{6Aqq}$ reducitur ad $\frac{2Aq}{3}$ dividendo per 2Aqq. Item $\frac{BA}{B}$ reducitur ad A, dividendo utrumque per B. Nam quod multiplicatio conficit, diviſio diſſolvit.

3 Quare ſi maxima duorum numerorum communis menſura ſit 1: dicuntur duo illi numeri primi inter ſe. Suntque minimi in eadem ratione, ut 29 & 24.

Si numerus primus ſit ad utrumque factorem, primus erit ad factum. Hinc proportionis operatio fieri ſæpenumero poteſt facilior, ut in exemplo.

$$\begin{array}{llll} 1 & & & \\ \cancel{8} & 2 & & 5 \\ \cancel{12}.\cancel{8} & :: & \cancel{15}. & 10. \end{array}$$

5 Memento autem diligenter, *Quotieſcunque fractio aliqua, ſive ratio, proponitur, ut ipſam primò ad minimos terminos reducas*, ut $\frac{744}{899}$ fiant $\frac{24}{29}$.

Cap. VIII.

DE PARTIBUS: quæ etiam fractiones, ſive numeri fracti, dicuntur.

1 VNitas (ſive integrum unum quodque) concipi mente poteſt in quotcunque æquales partes diviſibilis: quæ quidem partes denominationem ex numero ſuo, quem unitas continet, ſortiuntur: ut ſi unitas intelligatur dividi in binas æquales partes, dicuntur ſecundæ; ſi in tres, tertiæ: & ſic de reliquis.

2. Scribuntur partes duobus terminis cum lineola interjecta: quorum inferior denotat unitatem diviſam in totidem æquales partes; & dicitur denominator. Superior verò oſtendit quot ex partibus illis ſignificantur; atque ideò dicitur numerator. Ut $\frac{4}{5}$ numerator / denominator { & ſignificant quatuor quintas partes, ſive quatuor partes unius integri diviſi quinquifariam.

3 *Quam igitur rationem habet numerator ad denominatorem, eandem habet quantitas ſignificata ad unitatem.* 4.5 : : $\frac{4}{5}$.1. R.S : : $\frac{R}{S}$.1

4. Et quia ratio quævis terminis innumeris ſimiliter ſeſe adinvicem habentibus (quorum quidem maximi dari nequeunt) poterit exprimi: ſequitur partes etiam eaſdem, non iiſdem ſolummodò numeris, ſed aliis infinitis, poſſe deſignari. Ut quincuncem ſignificant

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cant non modo $\frac{5}{12}$, qui minimi ſunt termini in eadem ratione, ſed etiam $\frac{10}{24}$, $\frac{20}{48}$, $\frac{25}{60}$, $\frac{45}{108}$: & quotcunque alii numeri fiunt multiplicando 5 & 12 in alium quemvis numerum, per 2 cap. 6.

5 Quare æqualium partium, ſive fractionum, termini ſunt proportionales, & contra.

6 Item, ſi partium numerator minor ſit denominatore, partes ſunt unitate minores : ſi æqualis, ſignificant unitatem. Et ſi major, partes unitatem excedunt, eâdem ratione, quâ denominator à numeratore ſuperatur. Reducuntur autem ad unitates dividendo numeratorem per denominatorem : ut $\frac{31}{7}$ ſunt $4\frac{3}{7}$ item $\frac{CR+SA}{R}$ eſt $C+\frac{SA}{R}$ Et contra integri, ſive unitates reſolvuntur in partes cujuſque generis multiplicando unitates per denominatorem earundem partium, ut 1 fiet $\frac{2}{2}$, vel $\frac{3}{3}$, &c. & $4\frac{3}{7}$ fient $\frac{28+3}{7}$, hoc eſt $\frac{31}{7}$. item $C+\frac{SA}{R}$ fiet $\frac{CR+SA}{R}$

Cap. IX.

DE ADDITIONE ET SUBDUCTIONE PARTIUM.

1 SI partes propoſitæ diverſarum ſint ſpecierum : Primò reducendæ ſunt ad eandem denominationem

tionem, dividendo denominatores per maximam ipſorum communem menſuram; & multiplicando terminos per alternos quotos. Deinde in numeratoribus partium inventarum ejuſdem denominationis additio vel ſubductio inſtituenda eſt: Et ſummæ denique, vel differentiæ, communis ille denominator ſubſcribendus.

2 Et ſi integri partibus ſint immixti, ſeorſum tamen ſunt numerandi. Exempli gratia: Ex $6\frac{1}{18}$ tollatur $\frac{13}{16}$ & $2\frac{7}{12}$. Primo addendæ ſunt $\frac{13}{16}$ & $2\frac{7}{12}$ eruntque 2 & $\frac{39+28}{48}$ vel $\frac{67}{48}$, nempe $3\frac{19}{48}$: quibus demptis è $6\frac{1}{18}$ reſtabunt $2\frac{95}{144}$ ut in exemplo

```
     67                                  152
   39+28        57      8              5 ———
   13    7      19      1               144
  —— 2 ——  eſt 3 —— è 6 ——               57
4) 16   12      48     18              3 ——
    4   3     6) 8      3               144
    ⏟             ⏟                   ————
    48            144                  2 95/144 R
```

Adde $\frac{A}{B}$ & Z, ſumma $\frac{A+ZB}{B}$

```
  BE+DA
  B + D
C) CA  CE
   A    E
    ⏟
   CAE
```

Ex $\frac{A}{B}$ tolle $\frac{B}{C}$, reſtat $\frac{CA—Bq}{BC}$.

¶ Cap. X.

DE MULTIPLICATIONE ET DIVISIONE PARTIUM.

1 Multiplicatio comparat heterologos terminos (hoc eſt reducit ad minimos) & multiplicat homologos.

2 Diviſio comparat homologos terminos, & multiplicat heterologos.

3 Et ſi integri partibus ſint immixti, reſolvendi ſunt integri in partes.

Exempla Multiplicationis.

$\overset{1}{\frac{9}{16}}_{4}$ in $\overset{5}{\frac{20}{27}}_{3}$ fit $\frac{5}{12}$ | $\overset{4}{\frac{8}{9}}$ in $\frac{5}{6}_{3}$ fit $\frac{20}{27}$ | $\frac{5}{1}$ in $3\frac{1}{4}$ $\left(\overset{13}{\frac{13}{4}}\right)$ fit $\frac{65}{4}$ ($16\frac{1}{4}$

$\frac{A}{B}$ in B fit A | $\frac{A}{B}$ in Z fi $\frac{ZA}{B}$ | $\frac{A}{B}$ in $\frac{ZA}{C}$ fit $\frac{ZAq}{BC}$

Exempla Diviſionis.

$\overset{3}{\frac{9}{16}}_{4}\Big)\overset{5}{\frac{15}{28}}_{7}\Big(\frac{20}{21}$ | $\frac{8}{15}_{3}\Big)7\overset{37}{\frac{2}{5}}_{1}\Big(\frac{111}{8}\Big(13\frac{7}{8}$ | $\overset{1}{\frac{3}{4}}\Big)\overset{3}{\frac{9}{1}}\Big(\frac{12}{1}$

$\frac{D}{1}\Big)\frac{Aq}{B}\Big(\frac{Aq}{DB}$ | $\frac{A}{D}\Big)\frac{BC}{1}\Big(\frac{BCD}{A}$ | $\frac{A}{B}\Big)\frac{BC}{1}\Big(\frac{BqC}{A}$

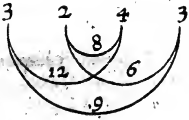
$\frac{B}{A}\Big)\frac{BC}{1}\Big(\frac{CA}{1}$ | $\frac{Ac}{C}\Big)\frac{Bc}{D}\Big(\frac{BcC}{DAc}$

4 Quis

4. Quis numerus eſt $\frac{2}{7}$ è 21? Multiplica 21 per $\frac{2}{7}$. Nam 1. $\frac{2}{7}$ :: 21.6. vel 7.2 :: 21.6.

5. Cujus numeri 6 continet $\frac{2}{7}$? Divide 6 per $\frac{2}{7}$. Nam $\frac{2}{7}$. 1 :: 6. 21. vel 2.7 :: 6.21.

6. Apud antiquos Muſicæ Scriptores, termini multiplicandi in rationum ſive continuatione, ſive imminutione, connectuntur lineolis curvis, in hunc modum.



7. Rationum continuatio fit per Multiplicationem ipſarum, ac ſi eſſent fractiones. Continuentur rationes 3 ad 2, & 4 ad 3: idem eſt ac ſi dicatur, multiplicentur $\frac{3}{2}$ in $\frac{4}{3}$, fientque $\frac{12}{6}$, quæ dupla eſt ratio. Quare ratio ſeſquialtera continuata cum ratione ſeſquitertiâ facit duplum: vel ut loquuntur Muſici, ex diapente & diateſſaron fit diapaſon.

Rationum imminutio fit per Diviſionem: ut è ratione 3 ad 2 detrahenda ſit 4 ad 3: Idem eſt ac ſi jubeatur $\frac{3}{2}$ dividi per $\frac{4}{3}$ reſtabitque $\frac{9}{8}$: nam $\frac{4}{3}$) $\frac{3}{2}$ ($\frac{9}{8}$, ratio ſeſquioctava: quæ menſura eſt Toni integri. Unde dicunt Muſici quod differentia inter diapente & diateſſaron eſt Tonus. Ut in hâc lineâ ſive chordâ diviſâ in duodecim partes.



CAP.

Cap. XI.

Exempla aliquot faciliima, quibus quæ hactenus tradita ſunt familiaria redduntur: Et via ad Æquationem Analyticam ſternitur.

1 SCiendum primò eſt, quod in ſequentibus, tum brevitatis, tum phantaſiæ juvandæ gratia paſſim fere his verborum ſymbolis utor. A & E ſignificant duos numeros, ſive magnitudines; quorum A plerumque major eſt, E minor. Æ rectangulum ſub ipſis. Z eſt ſumma. X differentia. Zq. ſummæ quadratum. Xq differentiæ quadratorum, Z ſumma quadratorum. X differentia quadratorum Z ſumma cuborum. X differentia cuborum. A, M, E, ſunt tres continuè proportionales: A, M, N, E, quatuor. Q: C: QQ: QC: &c. præfixæ magnitudinibus inter duo utrinq; puncta incluſis, ſignificant illiuſmodi poteſtates. √ denotat radicem ſive latus poteſtatis ſimplicis, ſi non intercedant duo puncta: Si vero poteſtas duobus utrinque punctis includatur, ſignificat latus ipſius univerſale: quod etiam aliter per literam b vel r deſcribi ſolet, ut √b latus eſt Binomii,& √r latus Reſidui ſive Apotomes. = nota eſt æqualitatis.

2. Sunt duo numeri ſive magnitudines, quorum major eſt A, minor E: quænam eſt ipſorum ſumma? quæ differentia? quod ſub ipſis rectangulum? quæ quadratorum ſumma? quæ quadratorum differentia?

quæ

quæ ſummæ & differentiæ ipſorum ſumma? quæ ſummæ & differentia ipſorum differentiæ? quod ſummæ & differentiæ ipſorum rectangulum? quod ſummæ quadratum? quod differentiæ quadratum? quæ quadratorum ſummæ & differentiæ ſumma? quæ quadratorum ſummæ & differentiæ differentia? quod quadratum rectanguli?

Z eſt A + E. X eſt A − E. Æ eſt AE.
Z = Aq + Eq. X = Aq − Eq.
Z + X = 2A. Z − X = 2E. ½ Z + ½ X = A &
ZX = Aq − Eq − X. (½ Z − ½ X = E
Zq = Aq + 2AE + Eq. Xq = Aq − 2AE + Eq.
Zq + Xq = 2Aq + 2Eq. Zq − Xq = 4AE: &
Æq = AqEq. (¼ Zq − ¼ Xq = AE:

3 Sunt duo numeri ſive magnitudines, quorum ſumma eſt Z, & major ex ipſis ponitur A: quiſnam eſt minor? quæ ipſorum differentia? quod ſubipſis rectangulum? quæ quadratorum ſumma? quæ quadratorum differentia?

E = Z − A. X = 2A − Z. Æ = ZA − Aq.
Z = Zq − 2ZA + 2Aq. X = 2ZA − Zq.
Si vero minor ex ipſis ponatur E:
A = Z − E. X = Z − 2E. Æ = ZE − Eq.
Z = Zq − 2ZE + 2Eq. X = Zq − 2ZE.

4 Sunt duo numeri ſive magnitudines, quorum differentia eſt X, & major ex ipſis ponitur A: quiſnam eſt minor? quæ ipſorum ſumma? quod ſub-

ipſis

ipſis rectangulum? quæ quadratorum ſumma? quæ quadratorum differentia?

$E = A - X$. $Z = 2A - X$. $Æ = Aq - XA$.
$Z = 2Aq - 2XA + Xq$. $X = 2XA - Xq$.

Si vero minor ex ipſis ponatur E:

$A = E + X$. $Z = 2EE +$. $Æ = Eq + XE$.
$Z = 2Eq + 2XE + Xq$. $X = 2XE + Xb$.

5. Sunt duo numeri ſive magnitudines, quorum major ad minorem, rationem habet R ad S; & major ex ipſis ponitur A: quiſnam eſt minor? quæ ipſorum ſumma? quæ ipſorum differentia? quod ſub ipſis rectangulum? quæ quadratorum ſumma? quæ quadratorum differentia.

$$E = \frac{SA}{R}. \quad Z = \frac{RA + SA}{R}. \quad X = \frac{RA - SA}{R}.$$

$$Æ = \frac{SAq}{R}. \quad Z = \frac{RqAq + SqAq}{Rq}. \quad X = \frac{RqAq - SqAq}{Rq}.$$

Si vero minor ex ipſis ponatur E:

$$A = \frac{RE}{S}. \quad Z = \frac{RE + SE}{S}. \quad X = \frac{RE - SE}{S}.$$

$$Æ = \frac{REq}{S}. \quad Z = \frac{RqEq + SqEq}{Sq}. \quad X = \frac{RqEq \; SqEq}{Sq}.$$

6. Sunt duo numeri ſive magnitudines, quorum rectangulum eſt Æ; & major ex ipſis ponitur A: quiſnam eſt minor? quæ ipſorum ſumma? quæ ipſorum differentia? quæ quadratorum ſumma? quæ quadratorum differentia?

$$E = \frac{Æ}{A} \qquad Z = \frac{Aq + Æ}{A}. \qquad X = \frac{Aq - Æ}{A}.$$

$$Z = \frac{Aqq + Æq}{E}. \qquad X = \frac{Aqq - AqEq}{Aq}$$

Si verò minor ex ipſis ponatur E:

$$A = \frac{Æ}{E}. \qquad Z = \frac{Æ + Eq}{E}. \qquad X = \frac{Æ - Eq}{E}$$

$$Z = \frac{Æq + Eqq}{Eq}. \qquad X = \frac{Æq - Eqq}{Eq}.$$

7. Atque ex his comparatis multæ æqualitates oriuntur. Exempla ſumemus in ſumma & Differentia.

$$Z = A + E = 2A - X = 2E + X = \frac{Aq + Æ}{A} = \frac{Æ + Eq}{E} \text{ \&c.}$$

$$X = A - E = 2A - Z = Z - 2E = \frac{Aq - Æ}{A} = \frac{Æ - Eq}{E} \text{ \&c.}$$

Hoc modo etiam in reliquis comparationes poterunt inſtitui, quibus eadem magnitudo multas admittet interpretationes atque diverſitates.

CAP.

Cap. XII.

DE GENESI, ET ANALYSI POTESTATUM.

1 Quia omnia reſolvuntur in eaſdem partes, ex quibus coagmentantur: primò ſcire oportet ex quibus partibus quælibet poteſtas conſtituitur. Poteſtates autem fiunt à radice aliquoties in ſe multiplicatâ. Nam latus in ſe ductum facit quadratum: Quadratum ductum in latus facit cubum. Cubus ductus in latus ſuum facit quadrato-quadratum, quæ poteſtas eſt quartana |4|: hæc iterum ducta in latus facit quadrato-cubum, ſcilicet quintanam |5|: Et ſic ulterius progrediendo fiunt poteſtates ſextana |6|, ſeptimana |7|, octavana |8|, nonana |9|, decumana |10|, & reliquæ, pro numero dimenſionum ſuarum, ex quibus componuntur.

Quare poteſtatum à radice ſingulari, quæ unica figura, ſive nota, conſtat, procreatio nihil habet difficultatis.

2 TABELLA PRIOR POTESTATUM A RADICE SINGULARI.

L N	2 q	3 c	4 qq	5 qc	6 cc	7 qqc	8 qcc
1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256
3	9	27	81	243	729	2187	6561
4	16	64	256	1024	4096	16384	63536
5	25	125	625	3125	15625	78125	390625
6	36	216	1296	7776	46656	279936	1679616
7	49	343	2401	16087	117649	823543	5764801
8	64	512	4096	32768	262144	2097152	16777216
9	81	729	6581	59049	531441	4782969	43046721

3 Quæ

3 Quæ verò à radice binarum notarum exurgunt, hunc habent ortus ſui modum.

Geneſis poteſtatum à radice binomia.

$$
\begin{array}{l}
A + E \\
A + E \\
\hline
Aq + AE \\
\quad + AE + Eq \\
\hline
Aq + 2AE + Eq. \quad \text{Quadratum} \\
A + E \\
\hline
Ac + 2AqE + AEq \\
\quad + AqE + 2AEq + Ec \\
\hline
Ac + 3AqE + 3AEq + Ec. \quad \text{Cubus} \\
A + E \\
\hline
Aqq + 3AcE + 3AqEq + AEc \\
\quad + AcE + 3AqEq + 3AEc + Eqq \\
\hline
Aqq + 4AcE + 6AqEq + 4AEc + Eqq. \quad \text{Quadrato-quadr:} \\
A + E \quad \&c
\end{array}
$$

4 Atque hoc artificio conficietur tabula poteſtatum adſcendentium in ſcala à radice binomia: quæ *POSTERIOR* vocetur.

A E

2	3	4	5	6	7	8	9	10
Aq	Ac	Aqq	Aqc	Acc	Aqqc	Aqcc	Accc	Aqqcc
2AE	3AqE	4AcE	5AqqE	6AqcE	7AccE	8AqqcE	9AqccE	10AcccE
Eq	3AEq	6AqEq	10AcEq	15AqqEq	21AqcEq	28AccEq	36AqqcEq	45AqccEq
	Ec	4AEc	10AqEc	20AcEc	35AqqEc	56AqcEc	84AccEc	120AqqcEc
		Eqq	5AEqq	15AqEqq	35AcEqq	70AqqEqq	126AqcEqq	210AccEqq
			Eqc	6AEqc	21AqEqc	56AcEqc	126AqqEqc	252AqcEqc
				Ecc	7AEcc	28AqEcc	84AcEcc	210AqqEcc
					Eqqc	8AEqqc	36AqEqqc	120AcEqqc
						Eqcc	9AEqcc	45AqEqcc
							Eccc	10AEccc

5 Quælibet ſpecies intermedia cujuſque ordinis componitur ex duabus ſpeciebus ordinis præcedentis utrinque proximis: nempe A poteſtate ſuperioris ſpeciei, & E poteſtate inferioris. Numerus etiam adfigendus ex utroque numero iiſdem adfixo aggregatur. Quare continuari facile poterit hæc tabula ulterius pro libitu.

6 In hâc tabulâ duæ extremæ poteſtates ſingulorum generum ſunt diagonales: & ſpecies intermediæ ſunt complementa: quibus adfixæ ſunt unciæ, oſtendentes numerum complementorum in conſtitutione cujuſque poteſtatis ſumendorum. Complementa autem omnia, cum E poteſtate, Gnomon non ineptè dici poterit.

7 Ex hâc tabulâ etiam liquet, quod quadratum à radice binarum notarum conſtat ex diagonalibus quadratis utriuſque notæ, & duplice rectangulo ſub ipſis notis. Cubus autem conſtat ex cubis diagonalibus, & triplice ſolido ſub quadrato majoris notæ & notâ minore, & triplice item ſolido ſub majore notâ & quadrato minoris. Quod ſimiliter de reliquis quoque poteſtatibus eſt efferendum.

8 Oſtendit inſuper plena hæc myſteriis pulcherrimis tabella, in numeroſa poteſtate, ſedes tum poteſtatum ſingularium ſive diagonalium, tum cujuſque ſpeciei complementorum. Nam cùm inter bina quadrata unica eſt ſpecies, quadratorum ſedes unicum interponent pro complementis locum. Et cùm inter binos cubos duæ ſunt complementorum ſpecies,

	2	3	4	5	6	7	8	9	10
A	Aq	Ac	Aqq	Aqc	Acc	Aqqc	Aqcc	Accc	Aqqcc
	2AE	3AqE	4AcE	5AqqE	6AqcE	7AccE	8AqqcE	9AqccE	10AcccE
E	Eq	3AEq	6AqEq	10AcEq	15AqqEq	21AqcEq	28AccEq	36AqqcEq	45AqccEq
		Ec	4AEc	10AqEc	20AcEc	35AqqEc	56AqcEc	84AccEc	120AqqcEc
			Eqq	5AEqq	15AqEqq	35AcEqq	70AqqEqq	126AqcEqq	210AccEqq
				Eqc	6AEqc	21AqEqc	56AcEqc	126AqqEqc	252AqcEqc
					Ecc	7AEcc	28AqEcc	84AcEcc	210AqqEcc
						Eqqc	8AEqqc	36AqEqqc	120AcEqqc
							Eqcc	9AEqcc	45AqEqcc
								Eccc	10AEccc
									Eqqcc

5 Qualibet ſpecies intermedia cujuſque ordinis componitur ex duabus ſpeciebus ordinis præcedentis utrinque proximis: nempe A poteſtate ſuperioris ſpeciei, & E poteſtate inferioris. Numerus etiam adfigendus ex utroque numero iiſdem adfixo aggregatur. Quare continuari facile poterit hæc tabula ulterius pro libitu.

6 In hâc tabulâ duæ extremæ poteſtates ſingulorum generum ſunt diagonales: & ſpecies intermediæ ſunt complementa: quibus adfixæ ſunt unciæ, oſtendentes numerum complementorum in conſtitutione cujuſque poteſtatis ſumendorum. Complementa autem omnia, cum E poteſtate, Gnomon non ineptè dici poterit.

7 Ex hâc tabulâ etiam liquet, quod quadratum à radice binarum notarum conſtat ex diagonalibus quadratis utriuſque notæ, & duplice rectangulo ſub ipſis notis. Cubus autem conſtat ex cubis diagonalibus, & triplice ſolido ſub quadrato majoris notæ & notâ minore, & triplice item ſolido ſub majore notâ & quadrato minoris. Quod ſimiliter de reliquis quoque poteſtatibus eſt efferendum.

8 Oſtendit inſuper plena hæc myſteriis pulcherrimis tabella, in numeroſa poteſtate, ſedes tum poteſtatum ſingularium ſive diagonalium, tum cujuſque ſpeciei complementorum. Nam cùm inter bina quadrata unica eſt ſpecies, quadratorum ſedes unicum interponent pro complementis locum. Et cùm inter binos cubos duæ ſunt complementorum ſpecies,

	2	3	4	5	6	7	8	9	10
A	Aq	Ac	Aqq	Aqc	Acc	Aqqc	Aqcc	Accc	Aqqcc
	2AE	3AqE	4AcE	5AqqE	6AqcE	7AccE	8AqqcE	9AqccE	10AcccE
E	Eq	3AEq	6AqEq	10AcEq	15AqqEq	21AqcEq	28AccEq	36AqqcEq	45AqccEq
		Ec	4AEc	10AqEc	20AcEc	35AqqEc	56AqcEc	84AccEc	120AqqcEc
			Eqq	5AEqq	15AqEqq	35AcEqq	70AqqEqq	126AqcEqq	210AccEqq
				Eqc	6AEqc	21AqEqc	56AcEqc	126AqqEqc	252AqcEqc
					Ecc	7AEcc	28AqEcc	84AcEcc	210AqqEcc
						Eqqc	8AEqqc	36AqEqqc	120AcEqqc
							Eqcc	9AEqcc	45AqEqcc
								Eccc	10AEccc
									Eqqcc

5 Quælibet ſpecies intermedia cujuſque ordinis componitur ex duabus ſpeciebus ordinis præcedentis utrinque proximis: nempe A poteſtate ſuperioris ſpeciei, & E poteſtate inferioris. Numerus etiam adfigendus ex utroque numero iiſdem adfixo aggregatur. Quare continuari facile poterit hæc tabula ulterius pro libitu.

6 In hâc tabulâ duæ extremæ poteſtates ſingulorum generum ſunt diagonales: & ſpecies intermediæ ſunt complementa: quibus adfixæ ſunt unciæ, oſtendentes numerum complementorum in conſtitutione cujuſque poteſtatis ſumendorum. Complementa autem omnia, cum E poteſtate, Gnomon non ineptè dici poterit.

7 Ex hâc tabulâ etiam liquet, quod quadratum à radice binarum notarum conſtat ex diagonalibus quadratis utriuſque notæ, & duplice rectangulo ſub ipſis notis. Cubus autem conſtat ex cubis diagonalibus, & triplice ſolido ſub quadrato majoris notæ & notâ minore, & triplice item ſolido ſub majore notâ & quadrato minoris. Quod ſimiliter de reliquis quoque poteſtatibus eſt efferendum.

8 Oſtendit inſuper plena hæc myſteriis pulcherrimis tabella, in numeroſa poteſtate, ſedes tum poteſtatum ſingularium ſive diagonalium, tum cujuſque ſpeciei complementorum. Nam cùm inter bina quadrata unica eſt ſpecies, quadratorum ſedes unicum interponent pro complementis locum. Et cùm inter binos cubos duæ ſunt complementorum ſpecies,

	2	3	4	5	6	7	8	9	10
								Accc	Aqqcc
							Aqcc	9AqccE	10AcccE
						Aqqc	8AqqcE	36AqqcEq	45AqccEq
					Acc	7AccE	28AccEq	84AccEc	120AqqcEc
				Aqc	6AqcE	21AqcEq	56AqcEc	126AqcEqq	210AccEqq
			Aqq	5AqqE	15AqqEq	35AqqEc	70AqqEqq	126AqqEqc	252AqcEqc
		Ac	4AcE	10AcEq	20AcEc	35AcEqq	56AcEqc	84AcEcc	210AqqEcc
A	Aq	3AqE	6AqEq	10AqEc	15AqEqq	21AqEqc	28AqEcc	36AqEqqc	120AcEqqc
	2AE	3AEq	4AEc	5AEqq	6AEqc	7AEcc	8AEqqc	9AEqcc	45AqEqcc
E	Eq	Ec	Eqq	Eqc	Ecc	Eqqc	Eqcc	Eccc	10AEccc
									Eqqcc

5 Quælibet ſpecies intermedia cujuſque ordinis componitur ex duabus ſpeciebus ordinis præcedentis utrinque proximis: nempe A poteſtate ſuperioris ſpeciei, & E poteſtate inferioris. Numerus etiam adfigendus ex utroque numero iiſdem adfixo aggregatur. Quare continuari facile poterit hæc tabula ulterius pro libitu.

6 In hâc tabulâ duæ extremæ poteſtates ſingulorum generum ſunt diagonales: & ſpecies intermediæ ſunt complementa: quibus adfixæ ſunt unciæ, oſtendentes numerum complementorum in conſtitutione cujuſque poteſtatis ſumendorum. Complementa autem omnia, cum E poteſtate, Gnomon non ineptè dici poterit.

7 Ex hâc tabulâ etiam liquet, quod quadratum à radice binarum notarum conſtat ex diagonalibus quadratis utriuſque notæ, & duplice rectangulo ſub ipſis notis. Cubus autem conſtat ex cubis diagonalibus, & triplice ſolido ſub quadrato majoris notæ & notâ minore, & triplice item ſolido ſub majore notâ & quadrato minoris. Quod ſimiliter de reliquis quoque poteſtatibus eſt efferendum.

8 Oſtendit inſuper plena hæc myſteriis pulcherrimis tabella, in numeroſa poteſtate, ſedes tum poteſtatum ſingularium ſive diagonalium, tum cujuſque ſpeciei complementorum. Nam cùm inter bina quadrata unica eſt ſpecies, quadratorum ſedes unicum interponent pro complementis locum. Et cùm inter binos cubos duæ ſunt complementorum ſpecies,

	2	3	4	5	6	7	8	9	10
A	Aq	Ac	Aqq	Aqc	Acc	Aqqc	Aqcc	Accc	Aqqcc
	2AE	3AqE	4AcE	5AqqE	6AqcE	7AccE	8AqqcE	9AqccE	10AcccE
E	Eq	3AEq	6AqEq	10AcEq	15AqqEq	21AqcEq	28AccEq	36AqqcEq	45AqccEc
		Ec	4AEc	10AqEc	20AcEc	35AqqEc	56AqcEc	84AccEc	120AqqcEc
			Eqq	5AEqq	15AqEqq	35AcEqq	70AqqEqq	126AqcEqq	210AccEqc
				Eqc	6AEqc	21AqEqc	56AcEqc	126AqqEqc	252AqcEqc
					Ecc	7AEcc	28AqEcc	84AcEcc	210AqqEc
						Eqqc	8AEqqc	36AqEqqc	120AcEqq
							Eqcc	9AEqcc	45AqEqc
								Eccc	10AEccc
									Eqqcc

5 Quælibet ſpecies intermedia cujuſque ordinis componitur ex duabus ſpeciebus ordinis præcedentis utrinque proximis: nempe A poteſtate ſuperioris ſpeciei, & E poteſtate inferioris. Numerus etiam adfigendus ex utroque numero iiſdem adfixo aggregatur. Quare continuari facile poterit hæc tabula ulterius pro libitu.

6 In hâc tabulâ duæ extremæ poteſtates ſingulorum generum ſunt diagonales: & ſpecies intermediæ ſunt complementa : quibus adfixæ ſunt unciæ, oſtendentes numerum complementorum in conſtitutione cujuſque poteſtatis ſumendorum. Complementa autem omnia, cum E poteſtate, Gnomon non ineptè dici poterit.

7 Ex hâc tabulâ etiam liquet, quod quadratum à radice binarum notarum conſtat ex diagonalibus quadratis utriuſque notæ, & duplice rectangulo ſub ipſis notis. Cubus autem conſtat ex cubis diagonalibus, & triplice ſolido ſub quadrato majoris notæ & notâ minore, & triplice item ſolido ſub majore notâ & quadrato minoris. Quod ſimiliter de reliquis quoque poteſtatibus eſt efferendum.

8 Oſtendit inſuper plena hæc myſteriis pulcherrimis tabella, in numeroſa poteſtate, ſedes tum poteſtatum ſingularium ſive diagonalium, tum cujuſque ſpeciei complementorum. Nam cùm inter bina quadrata unica eſt ſpecies, quadratorum ſedes unicum interponent pro complementis locum. Et cùm inter binos cubos duæ ſunt complementorum ſpecies,

cuborum ſedes binos interponent locos complementis ſuis ordine diſtribuendos.

Cap. XIII.

His itaque præmiſſis ad GENESIN poteſtatum accedamus.

1 PRoponatur Geneſis quadrati à latere 57. major igitur nota A eſt 5, minor E eſt 7. Scribantur 5 & 7 intermiſſo unius gradus ſpacio: & linea ſub ipſis ducatur. Sub 5 ſtatuatur quadratum ſuum 25: & ſub 7 ſuum 49. tum duplicetur 5, & multiplicetur per 7, fietque duplum rectangulum 70, ponendum loco intermedio. addantur omnia ſuis quæque locis:

5	7		
25		Aq	
70		2AE	gnomon
	49	Eq	
3249			

ſumma erit 3249 pro quadrato lateris 57 quæſito.

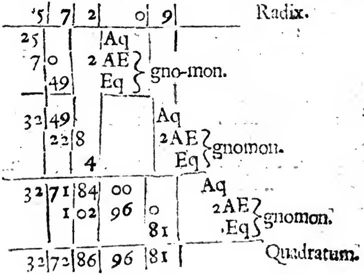
2 Proponatur iterum Geneſis cubi à latere 57. ſcribantur 5 & 7 intermiſſo duorum graduum ſpacio: & linea ſub ipſis ducatur. ſub 5 ſtatuatur cubus ſuus 125: & ſub 7 ſuus 343. tum quadratum à 5

5	7		
125		Ac	
525		3AqE	
735		3AEq	gnomon.
	343	Ec	
185193			

triplicetur, & multiplicetur per 7, fietque triplum ſolidum majus 525, ponendum loco priore intermedio:

dio : item triplicetur 5, & multiplicetur per 49 quadratum à 7, fietque triplum ſolidum minus 735, ponendum loco intermedio ſecundo. addantur omnia ſuis quæque locis : ſumma erit 185193 pro cubo lateris 57 quæſito.

3 Si latus propoſitum conſtet pluribus figuris, ut 57209: Primò poteſtas duarum primarum figurarum 47 quærenda eſt. Deinde ſumptis 57 pro A, & figura 2 ſequente pro E : quæratur poteſtas ipſius eodem, qui ante oſtenſus eſt tabellæ ordine. Quod etiam in reliquis figuris ſingulatim eſt faciendum.



5	7	2	0	9	Radix.	
125		Ac				
52	5	3AqE				
7	35	3AEq			gnomon	
	343	Ec				
185	193			Ac		
1	949	4		3AqE		
	6	84		3AEq	gnomon	
		8		Ec		
187	149	248	000	.	Ac	
	88	339	680	0	3AqE	
		13	899	60	3AEq	gnomon.
				729	Ec	
187	237	601	580	329	Cubus.	

4 Ex his, quæ jam declarata ſunt, non difficile erit reliquas etiam omnes ſuperiorum generum poteſtates progignere: modò in ipſarum geniturâ inferiorum omnium ad ipſis adſcendentium poteſtatum geneſis inſtituatur: ſicut in cubi geneſi jam factum vides.

CAP.

Cap. XIV.

Sequitur ANALYSIS: *quæ eſt eductio radicis ex numeroſa poteſtate data.*

1 ANalyſis, poſtquam ſedes poteſtatum, pro ſuo quaſque juxta tabulam genere, punctis, poſito primo puncto ſub loco unitatum, diſtinxerit: primò ex figuris primi à ſiniſtra puncti poteſtatem diagonalem comprehenſam tollit: latuſque ipſius, quod A vocetur in margine ſcribit. tum numero reliquo, ad proximum uſque punctum (qui gnomonem intelligitur continere) per diviſorem ex latere A invento legitimè conflatum, diviſo, ſecundum latus E quærit & in margine ſcribit: per quod demùm gnomonem perficit: perfectumque ex reliquo illo ſubtrahit. Et ſic integra duorum primorum ſingularium laterum, in duobus primis punctis contenta, poteſtate dempta, reſtabit ad tertium uſque punctum gnomon pro tertio latere ſimiliter eruendo.

Analyſis

Analyſis quadrati.

1			
723	02		
3272	8696	81	(57209
25	Aq		punctatio
10	2A	diviſor	
70	2AE }		
49	Eq }		
749		gnomon	
11	4 2A	diviſor	
22	8	2AE }	
	4	Eq }	
22	84	gnomon	
1	144	2A diviſor	
	1 1440	2A diviſor	
1	02960	2AE }	
		81 Eq }	
1	0296	81 gnomon.	

Analyſis Cubi.

```
      2|088
     62|0443 53
    187|2376 01|580|329   (57209
    125|  Ac
      75|    |3Aq  }
        |15  |3A   }
      7|65  divisor
    525|5    |3 AqE }
      7|35   |3 AEq }
       |343  |  Ec  }
     60|193 | gno-mon
       |9747       3Aq
       |   1|7 1   3A
       |976|41   divi-sor
      1|949|4      3 AqE
       |   6|84    3 AEq
       |     |8      Ec
    1956|248  gno-mon
       |98 |1552    3Aq }
       |     1716   3A  }
       |9817236    divisor
       |98 155200|0   3Aq
       |    171  |60  3A
       |9|8 156 91|60 divisor
       |883 968 00|0   3AqE }
       |    13899 |60  3AEq }
       |          |729   Ec }
       |883|353|3508|329 gnomon.
```

```
   5 |  7 | 2  0
  25 |
   70|
     |49
  32 |49
     |22 |8
     |   |  4
  32 |71 |84 00
```

2 Si numerus propoſitus non ſit verus ſui generis figuratus, ſed peracta Analyſi aliquid reſtet : punctationes circulorum pro ſuo genere, quot opus erit ſtatuendæ ſunt : & continuanda Analyſis poſt lineam ſeparatricem.

3 Ex his etiam quæ declarata ſunt, non difficile erit ope tabellæ radices ex ſuperioribus poteſtatibus omnibus educere.

Cap. XV.

De lateribus ſurdis.

1 SI quotlibet numeri ſint continuè proportionales. Erit ut primus ad ultimum, ſic poteſtas primi æquimultiplicati numero terminorum minus uno, ad poteſtatem ſimilem ſecundi, ſunto quatuor

∺A,M,N,E Quia	A. M::A.M M.N::A.M N. E::A.M	Erit per Multiplicationē A.E::Ac.Mc.

2 Numeri plani vel ſolidi ſimiles ſunt, quorum latera homologa ſunt proportionalia.

3 Numeri plani ſimiles ſunt in duplicatâ ratione (hoc eſt, ut quadrata) homologorum laterum. Sunt igitur numeri plani ſimiles, ut quadratus numerus ad quadratum numerum. Item numeri ſolidi ſimiles ſunt in triplicatâ ratione (hoc eſt, ut Cubi) homologorum

logorum laterum. Sunt igitur numeri ſolidi ſimiles, ut numerus Cubicus ad numerum Cubicum.

4 Et generaliter omnes figurati ſimiles plurium dimenſionum, ſunt in ratione homologorum laterum, æquimultiplicatâ numero dimenſionum, ex quibus componuntur. Dimenſiones ſunto quatuor; nempe ABCD unius, & EFGH alterius, in ratione R ad S.

Quia { A. E :: R.S; B. F :: R.S; C. G :: R.S; D.H :: R.S } Erit per multiplicationem ABCD.EFGH :: Rqq.Sqq.

5. Si numerus non ſit verus ſui generis figuratus, latus ejus dicitur ſurdum. & ſic notatur, √q6, √c4, √qq20, √qc13 : hoc eſt latus quadrati 6, latus cubi 4, latus quadrato-quadrati 20, latus quadrato-cubi 13.&c.

6 Latera ſurda commenſurabilia ſunt, quorum numeri ad minimos terminos reducti fiunt veri ſui generis figurati : ſuntque idcirco ut numerus ad numerum, ut √q12 & √q147 reducta ad minimos terminos per √q3 maximam utriuſque communem menſuram, fiunt √q4 & √q49, hoc eſt 2 & 7: quare cum √q12 & √q147 ſint ut 2 ad 7, erunt commenſurabilia. Sic √c40 & √c1715 ſunt ut 2 ad 7, quoniam diviſa per maximam ſuam communem menſuram √c5, fiunt √c8 & √c343; ideoque commenſurabilia.

7 Addun-

7 Adduntur autem, atque ſubtrahuntur, latera ſurda commenſurabilia, ſi ſummæ, vel differentiæ, numerorum ipſis ſimilium inventorum homogenea poteſtas ducatur in communem ipſorum menſuram. Ut $\sqrt{q147}+\sqrt{q12}$ eſt $\sqrt{q243}$; hoc eſt latus quadrati à $7+2$ (nempe 81) ductum in $\sqrt{q3}$ maximam ipſorum communem menſuram. Et $\sqrt{q147}-\sqrt{q12}$ eſt $\sqrt{q75}$; hoc eſt latus quadrati à $7-2$ (nempe 25) ductum etiam in $\sqrt{q3}$.

Item $\sqrt{c1715}+\sqrt{c40}$ eſt $\sqrt{c3645}$, hoc eſt latus cubi $7+2$ (nempe 729) ductum in $\sqrt{c5}$ maximam ipſorum communem menſuram. Et $\sqrt{c1715}-\sqrt{c40}$ eſt $\sqrt{c625}$; hoc eſt latus cubi à $7-2$ ductum etiam in $\sqrt{c5}$.

Additionis & ſubductionis operatio talis eſt.

$\sqrt{q3})\ \sqrt{q147}\ (\sqrt{q49}.\ 7$ $\sqrt{c5})\ \sqrt{c1715}\ (\sqrt{c343}.\ 7$

$\sqrt{q}\ 12\ (q4.\ 2.$ $\sqrt{c}\ 40\ (\sqrt{c}\ 8.\ 2$

$\sqrt{q243}\ \sqrt{q81}.\ 9$ ſumma $\sqrt{c3645}\ \sqrt{c729}.\ 9$

$\sqrt{q}\ 75\ \sqrt{q25}.\ 5$ diffr. $\sqrt{c625}\ \sqrt{c125}.\ 5$

$\sqrt{12}+\sqrt{\frac{27}{4}}$

vel

$\sqrt{\frac{48}{4}}+\sqrt{\frac{27}{4}}$

$\sqrt{3})\ \sqrt{48}\ (\sqrt{16}.\ 4$

$\sqrt{27}\ (\sqrt{9}:\ 3$

$\sqrt{\frac{147}{4}}:\ \sqrt{49}.\ 7$

$\sqrt{\frac{245}{12}}+\sqrt{\frac{5}{12}}$

$\sqrt{5})\ \sqrt{245}\ (\sqrt{49}.\ 7$

$\sqrt{5}\ (\sqrt{1}.\ 1$

$\sqrt{\frac{320}{12}}:\ \sqrt{64}.\ 8$

$\sqrt{\frac{180}{12}}:\ \sqrt{36}.$

8 Latera verò ſurda incommenſurabilia, atque hete-

heterogenea, adduntur, vel ſubtrahuntur, ſignis + vel — ut $\sqrt{q7} + \sqrt{q4}$. & $\sqrt{c10} - \sqrt{c5}$.

9 Si numerus figuratus per numerum figuratum homogeneum multiplicetur, factus erit numerus ejuſdem generis figuratus, cujus latus æquale eſt facto à lateribus numerorum multiplicatorum. Et ſi numerus figuratus per numerum figuratum homogeneum dividatur, quotus erit numerus ejuſdem generis figuratus, cujus latus æquale eſt quoto lateris majoris diviſi per latus minoris. Ut factus à numeris cubicis 343 & 27 eſt 9261, numerus etiam cubicus, cujus latus eſt 7×3. Item $\sqrt{q \frac{AqEq}{Eq}}$ eſt $\frac{AE}{B}$.

10 Quare laterum ſurdorum homogeneorum multiplicatio, & diviſio, procreat latus etiam ſurdum homogeneum: ut $\sqrt{q7}$ in $\sqrt{q3}$ eſt $\sqrt{q21}$. Et $\sqrt{q7}$) $\sqrt{q21}$ ($\sqrt{q3}$: vel $\sqrt{q \frac{21}{7}}$ eſt $\sqrt{q3}$. Item $\sqrt{qA}$ in $\sqrt{qE}$ eſt $\sqrt{qAE}$. Et $\sqrt{qA}$) $\sqrt{qAE}$ ($\sqrt{qE}$: vel $\sqrt{q \frac{AE}{A}}$ eſt $\sqrt{qE}$.

11 Latera verò heterogenea non multiplicantur, vel dividuntur, niſi prius ad idem genus reducuntur, quod fit dividendo indices utriuſque poteſtatis propoſitæ per maximam ipſorum communem menſuram: & multiplicando tum Indices per alternos quotos: tum ipſas poteſtates in ſpecies alternis quotis cognomines. Ut, ſi ad multiplicandum vel dividendum, proponantur $\sqrt{qq10}$ & $\sqrt{cc7}$. Primò reducuntur ad $\sqrt{cccc1000}$, & $\sqrt{cccc49}$: cubando 10, & quadrando 7. Tum demùm fiat multiplicatio, vel di-

viſio. Sic etiam $\surd qqA$, & $\surd ccBq$ reducuntur ad $\surd ccccAc$, & $\surd ccccBqq$: uti planiùs adparebit per praxim, quæ hic adponitur.

$$\surd\boxed{12}\,1000 \quad \surd\boxed{12}\,49 \qquad \surd\boxed{12}\,Ac \quad \surd\boxed{12}\,Bqq$$

$$\boxed{2})\ \surd\frac{\boxed{4}}{\boxed{2}}\,10 \quad \surd\frac{\boxed{6}}{\boxed{3}}\,7 \qquad \boxed{2})\ \surd\frac{\boxed{4}}{\boxed{2}}\,A \quad \surd\frac{\boxed{6}}{\boxed{3}}\,Bq$$

Rurſus ſi $\surd c32$ duplicandum ſit, vel multiplicandum per 2: pro 2 ſumatur $\surd c8$: & per ipſum multiplicetur $\surd c32$; fietque $\surd c256$, æquivalens bis $\surd c32$.

Item ſi dimidiandum ſit $\surd c32$, vel dividendum per 2 : pro 2 ſumatur $\surd c8$: & per ipſum dividatur $\surd c32$; orieturque $\surd c\frac{12}{8}$; hoc eſt $\surd c4$, æquivalens $\frac{1}{2}\surd c32$.

Sic etiam $\frac{22}{7}\surd qAq$, fiet $\surd q\frac{484}{49}Aq$, hoc eſt $\frac{22}{7}A$.

12 Si latus poteſtatis multiplicandum ſit ſecundùm exigentiam ſuæ ſpeciei : deleatur nota ſpeciei lateralis. ut Q: $\surd q64$, vel C: $\surd c64$, eſt 64.

13. Et ſi latus poteſtatis, cujus index eſt numerus compoſitus, multiplicandum ſit ſecundùm exigentiam alterutrius ſpeciei componentis : latus alterius ſpeciei numero ſpeciali ſolùm præfigatur : ut Q: $\surd cc64$ eſt $\surd c64$. & C: $\surd cc64$ eſt $\surd q64$. Nam $\surd cc$ eſt $\surd\,\boxed{2*3}$

14 Si magnitudo plurium nominum, ducatur in ſeipſam

ſeipſam, cum uno ex ſuis ſignis mutato; expurgabitur unum nomen. Ut $3+\sqrt{5}+\sqrt{2}$ in $3+\sqrt{5}-\sqrt{2}$, fiet $12+\sqrt{180}$.

CAP. XVI.

DE ÆQUATIONE. & De quæſtionibus per Æquationem ſolvendis.

1 QUotieſcunque problema aliquod, ſive quæſtio, proponitur: Puta præſtitum eſſe quod poſtulatur: aptáque adhibita ratiocinatione, pro quæſita magnitudine ponatur A, vel alia aliqua vocalis: pro magnitudinibus autem datis conſonantes: quò facilius magnitudines datæ ab incertis dignoſcantur.

2 Deinde magnitudines, tam datæ, quam quæſitæ, ſecundum conditionem quæſtioni convenientem, efformentur atque comparentur; addendo, ſubtrahendo, multiplicando, & dividendo donec tandem aliquid inveniatur magnitudini, de qua quæritur, vel ſuæ, ad quam adſcendet, poteſtati æquale.

3 Et quia in omni ferè æquatione, ubi primò ex involucris quæſtionis effulget, nota cum ignotis confunduntur: termini ipſius ità ſunt ordinandi, ut quæ in data habentur menſura, faciant unam partem, & quæ ignota quæruntur, alteram. Quod quo artificio fiat, regulæ quinque ſequentes commonſtrabunt.

4 Primò ſi magnitudo quæſita, vel aliquis ejus gradus, ſit in fractione: fiat omnium magnitudinum ad unam denominationem reductio: ut omiſſo communi illo denominatore, in ſolis numeratoribus æquatio cenſeatur. Ut $A-C=\frac{Aq+Bq}{D}+B+C$: Erit $DA-DC=Aq+Bq+DB+DC$.

5 Secundò, ſi, quæ in data habentur menſura, immiſceantur cum quæſitis: *fiat tranſpoſitio magnitudinum ex una parte in aliam ſub contrario ſigno.* Ut $DA-DC=Aq+Bq+DB+DC$: Et tranſpoſitis DC & Aq, erit $DA-Aq=2DC+DB+Bq$. Quæ etiam regula in omni tranſpoſitione ſervanda eſt.

6 Tertiò, ſi ſpecies altiſſima quæſitæ magnitudinis ducatur in magnitudinem aliquam datam: fiat omnium magnitudinum æquationis ad illam communis adplicatio. Ut $BAq+BqA=Zc$: erit $Aq+BA=\frac{Zc}{B}$.

7 Quartò, ſi contingat omnes datas magnitudines duci in gradum aliquem magnitudinis quæſitæ: fiat omnium, per adplicationem ad minimam ſpeciem, ſecundum ordinem tabellæ communis depreſſio. Ut $Aqq+BAc=ZqAq$, erit $Aq+BA=Zq$, expuncto in ſingulis Aq. Atque hoc modo æquatio quælibet propoſita poterit deprimi, ſive reduci ad minores ſpecies; Si terminorum omnium fiat ad eundem gradum communis adplicatio. Ut $Ac+XAq=Nc$, diviſa per A, fiet $Aq+XA=\frac{Nc}{A}$: at diviſa per Aq, fiet

A

$A+X=\frac{Nc}{Aq}$ Quæ quidem operatio in numeroſa adfectarum æquationum reſolutione uſus erit non contemnendi: quia latus quæſitum faciliùs æſtimatur in minoribus poteſtatibus, quam in majoribus.

8 Quintò, ſi magnitudo aliqua ſit latus ſurdum: æquatio in ipſis poteſtatibus eſt inſtituenda. Ut $\sqrt{q}BA+B=C$: vel per tranſpoſitionem $\sqrt{q}BA=C-B$. Ideòque ipſorum quadrata, $BA=Cq-2CB+Bq$: vel $A=\frac{Cq-2CB+Bq.}{B}$

Item $\sqrt{u}: BA+CA: -D=B$. Vel $\sqrt{u}: BA+CA:=D+B$. Ideòque & ipſorum quadrata $BA+CA=Bq+2BD+Dq$: vel $A=\frac{Bq+2BD+Dq.}{B+C}$

Denique $\sqrt{q}\frac{A}{3}=\sqrt{c}2A$: vel per 11 c15, $\sqrt{qc}\frac{Ac}{27}=\sqrt{qc}4Aq$. quare $Ac=108Aq$. Et $A=108$.

9 Æquationum, in quibus ſunt tres ſpecies æqualiter in ordine ſcalæ adſcendentes, conſtitutio liquebit ex ſect: 2, 3, 4, capitis 11: Nam quia

$Z-A=E$: ducatur utraque pars in A.
$Z-E=A$: ducatur utraque pars in E.
$A-X=E$: ducatur utraque pars in A.
$E+X=A$: ducatur utraque pars in E.

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5 Secundò, ſi, quæ in data habentur menſura, immiſceantur cum quæſitis: *fiat tranſpoſitio magnitudinum ex una parte in aliam ſub contrario ſigno.* Ut $DA - DC = Aq + Bq + DB + DC$: Et tranſpoſitis DC & Aq, erit $DA - Aq = 2DC + DB + Bq$. Quæ etiam regula in omni tranſpoſitione ſervanda eſt.

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A

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A

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Atque hac multiplicatione hujuſmodi orientur æquationes.

ZA-Aq=Æ	Aq-XA=Æ
ZAq-Aqq=Æq	Aqq-XAq=Æq
ZAc-Acc=Æc	Acc-XAc=Æc
&c	&c
ZE-Eq=Æ	Eq+XE=Æ
ZEq-Eqq=Æq	Eqq+XEq=Æq
ZEc-Ecc=Æc	Ecc+XEc=Æc
&c	&c

Quotieſcunque igitur proponitur Æquatio conſtans ex tribus ſpeciebus æqualiter in ordine ſcalæ adſcendentibus: Cogitabis magnitudinem abſolutam datam eſſe rectangulum ſub duabus magnitudinibus quæſitis, ſive latera ſint, ſive quadrata, ſive Cubi, &c: qualis ſcil: eſt poteſtas mediæ ſpeciei. In media autem ſpecie, ſi altiſſima ſpecies ſit negata, coëfficientem eſſe ſummam magnitudinum quæſitarum; Et de utrâque exponi. At ſi altiſſima ſpecies ſit adfirmata, coëfficientem eſſe magnitudinum quæſitarum differentiam; ipſam autem ſpeciem exponi de majore, negatam; vel de minore, adfirmatam.

Tum datis binarum magnitudinum ſumma & rectangulo, datur earundem differentia: vel data differentia & rectangulo, datur ſumma. Nam per 2 Cap: XI.

Q

$Q{:}\frac{1}{2}Z{:}-Æ = Q{:}\frac{1}{2}X$ $\}$ quare $\{$ $\sqrt{u}{:}\frac{1}{4}Zq - Æ{:} = \frac{1}{2}X.$
$Q{:}\frac{1}{2}X{:}+Æ = Q{:}\frac{1}{2}Z$ $\}$ $\{$ $\sqrt{u}{:}\frac{1}{4}Xq + Æ{:} = \frac{1}{2}Z.$

Denique datis binarum magnitudinum $\frac{1}{2}Z$ & $\frac{1}{2}X$, dantur ipſæ magnitudines; hiſce duabus Regulis.

I Reg. $\frac{1}{2}Z \pm \sqrt{u}{:}\,\frac{1}{4}Zq - Æ{:}(\frac{1}{2}X) = \frac{A.}{E.}$

II Reg. $\sqrt{u}{:}\,\frac{1}{4}Xq + Æ{:}(\frac{1}{2}Z) \pm \frac{1}{2}X = \frac{A.}{E.}$

10. *GENESIS* ſex Binomiorum ex lateribus ſuis ſurdis. Regula eſt, $Z + 2Æ = Zq$.
In Apotomis vero, $Z - 2Æ = Xq$.

Exempl: I. Quadretur Binomium $4 + \sqrt{11}$. Hîc Z eſt $16 + 11$, hoc eſt 27. Et Æ eſt $\sqrt{16} \times \sqrt{11}$, hoc eſt $\sqrt{176}$: cujus duplum eſt $\sqrt{704}$. Quadratum igitur erit $27 + \sqrt{704}$. Quod dicitur Binomium I.

Exempl: II. Quadretur Bimediale prius, $\sqrt{qq}\,12 + \sqrt{qq}\,\frac{27}{4}$. Hîc Z eſt $\sqrt{12} + \sqrt{\frac{27}{4}}$, vel $\sqrt{\frac{48}{4}} + \sqrt{\frac{27}{4}}$; hoc eſt $\sqrt{\frac{147}{4}}$, per 7, Cap: XV. Et Æ eſt $\sqrt{qq}\,12 \times \sqrt{qq}\,\frac{27}{4}$, vel $\sqrt{qq}\,3 \times \sqrt{qq}\,27$; hoc eſt, $\sqrt{qq}\,81$, ſcil: 3: cujus duplum eſt 6. Quadratum igitur erit $\sqrt{\frac{147}{4}} + 6$: Quod dicitur Binomium II.

Exempl: III. Quadretur Bimediale poſterius $\sqrt{qq}\,\frac{80}{3} + \sqrt{qq}\,15$. Hîc Z eſt $\sqrt{\frac{80}{3}} + \sqrt{15}$, vel $\sqrt{\frac{80}{3}} + \sqrt{\frac{45}{3}}$; hoc eſt $\sqrt{\frac{245}{3}}$, per 7, Cap. XV. Et Æ eſt $\sqrt{qq}\,\frac{80}{3} \times \sqrt{qq}\,15$, vel $\sqrt{qq}\,80 \times \sqrt{qq}\,5$; hoc eſt, $\sqrt{qq}\,400$, ſcil: $\sqrt{20}$: cujus duplum eſt $\sqrt{80}$. Quadratum igitur erit $\sqrt{\frac{245}{3}} + \sqrt{80}$. Quod dicitur Binomium III.

In tribus reliquis, quæ conſtant ex radicibus Bino-

mii & Reſidui connexis, ut $\sqrt{b}{:}A+E{:}\ pl\ \sqrt{r}{:}A-E{:}$ perſpicuum eſt Z eſſe 2A: & Æ eſſe $\sqrt{:}\ Aq-Eq$. quare

Exempl: IV. Quadretur Major, $\sqrt{b}{:}\frac{7}{2}+\sqrt{\frac{29}{4}}{:}\ pl\ \sqrt{r}{:}\frac{7}{2}-\sqrt{\frac{29}{4}}$. Hîc Z eſt $\frac{7}{2}+\frac{7}{2}$, hoc eſt, 7. & Æ eſt $\sqrt{u}{:}\frac{49}{4}-\frac{29}{4}{:}$ hoc eſt, $\sqrt{\frac{20}{4}}$, ſcil: $\sqrt{5}$: cujus duplum eſt $\sqrt{20}$. Quadratum igitur erit $7+\sqrt{20}$. Quod dicitur Binomium IV.

Exempl: V. Quadretur Potens rationale cum mediali, $\sqrt{b}{:}\sqrt{5}+1{:}\ pl\ \sqrt{r}{:}\sqrt{5}-1$. Hîc Z eſt $\sqrt{5}+\sqrt{5}$; hoc eſt, $\sqrt{20}$. Et Æ eſt $\sqrt{:}5-1{:}$ hoc eſt, $\sqrt{4}$, ſcil: 2: cujus duplum eſt 4. Quadratum igitur erit $\sqrt{20}+4$. Quod dicitur Binomium V.

Exempl: VI. Quadretur Potens duo medialia, $\sqrt{b}{:}\sqrt{5}+\sqrt{3}{:}\ pl\ \sqrt{r}{:}\ \sqrt{5}-\sqrt{3}$. Hîc Z eſt $\sqrt{5}+\sqrt{5}$, hoc eſt, $\sqrt{20}$. Et Æ eſt $\sqrt{:}5-3{:}$ hoc eſt, $\sqrt{2}$: cujus duplum eſt $\sqrt{8}$. Quadratum igitur erit $\sqrt{20}+\sqrt{8}$. Quod dicitur Binomium VI.

11. Analyſis. In Binomio igitur quadratico, majus nomen eſt Z: & minus nomen 2Æ. At in 2 Cap. XI, ordinatum eſt, $\frac{1}{4}Zq-Æ = \frac{1}{4}Xq$: ſcil: $\frac{1}{4}Q{:}A+E{:}-Æ = \frac{1}{4}Q{:}A-E$. Quare ſi pro A & E ſumantur ipſarum quadrata Aq & Eq, erit $\frac{1}{4}Q{:}\ Aq+Eq{:}-AqEq = \frac{1}{4}Q{:}\ Aq-Eq{:}$ hoc eſt, $\frac{1}{4}Zq-Æq = \frac{1}{4}Xq$, ex quo Theoremate pro Analyſi Binomii deducitur hæc Regula.

$$\frac{1}{2}Z \pm \sqrt{q}{:}\frac{1}{4}Zq-Æq{:}\left(\frac{1}{2}X\right) = \begin{matrix}Aq.\\ Eq.\end{matrix}$$

Exempl: I. Quæratur latus Binomii I, $27+\sqrt{704}$: nempe $Z+2Æ$. Quare $\frac{1}{2}Z$ eſt $\frac{27}{2}$ & Æ eſt $\sqrt{\frac{704}{4}}$: & $\frac{1}{4}Zq-Æq$ eſt $\frac{729}{4}-\frac{704}{4}$; hoc eſt, $\frac{25}{4}$: cujus latus $\frac{5}{2}$ eſt $\frac{1}{2}X$. At per

per Reg: $\frac{27}{2} \pm \frac{5}{2} = \begin{matrix} 16. & 4 \\ 11. & \sqrt{11} \end{matrix} \Big\}$ Latus igitur quæſitum eſt $4 + \sqrt{11}$. Et dicitur Binomium.

Exempl. II. Quæratur latus Binomii II, $\sqrt{\frac{147}{4}} + 6$: nempe $Z + 2Æ$. Quare $\frac{1}{2}Z$ eſt $\sqrt{\frac{147}{16}}$: Et $Æ$ eſt 3. & $\frac{1}{4}Zq - Æq$ eſt $\frac{147}{16} - (9)\frac{144}{16}$; hoc eſt $\frac{3}{16}$: cujus latus $\sqrt{\frac{3}{16}}$ eſt $\frac{1}{2}X$. At per Reg: $\sqrt{\frac{147}{16}} \pm \sqrt{\frac{3}{16}} = \begin{matrix} \sqrt{12}. & \sqrt{qq\,12}. \\ \sqrt{\frac{27}{4}}. & \sqrt{qq\,\frac{27}{4}}. \end{matrix} \Big\}$ Latus igitur quæſitum eſt $\sqrt{qq\,12} + \sqrt{qq\,\frac{27}{4}}$. Et dicitur Bimediale prius.

Exempl: III. Quæratur latus Binomii III, $\sqrt{\frac{245}{3}} + \sqrt{80}$: nempe $Z + 2Æ$. Quare $\frac{1}{2}Z$ eſt $\sqrt{\frac{245}{12}}$: & $Æ$ eſt $\sqrt{20}$. & $\frac{1}{4}Zq - Æq$ eſt $\frac{245}{12} - (20)\frac{240}{12}$; hoc eſt $\frac{5}{12}$: cujus latus $\sqrt{\frac{5}{12}}$ eſt $\frac{1}{2}X$. At per Regul: $\sqrt{\frac{245}{12}} \pm \sqrt{\frac{5}{12}} = \begin{matrix} \sqrt{\frac{80}{3}}. & \sqrt{qq\,\frac{80}{3}} \\ \sqrt{15}. & \sqrt{qq\,15} \end{matrix} \Big\}$ Latus igitur quæſitum eſt $\sqrt{qq\,\frac{80}{3}} + \sqrt{qq\,15}$. Et dicitur Bimediale poſterius.

Exempl: IV. Quæratur latus Binomii IV, $7 + \sqrt{20}$; nempe $Z + 2Æ$. Quare $\frac{1}{2}Z$ eſt $\frac{7}{2}$: & $Æ$ eſt $\sqrt{5}$: & $\frac{1}{4}Zq - Æq$ eſt $\frac{49}{4} - (5)\frac{20}{4}$; hoc eſt, $\frac{29}{4}$: cujus latus $\sqrt{\frac{29}{4}}$ eſt $\frac{1}{2}X$. At per Reg: $\frac{7}{2} \pm \sqrt{\frac{29}{4}} = \begin{matrix} \frac{7}{2} + \sqrt{\frac{29}{4}} & \sqrt{b:\frac{7}{2} + \sqrt{\frac{29}{4}}} \\ \frac{7}{2} - \sqrt{\frac{29}{4}} & \sqrt{r:\frac{7}{2} - \sqrt{\frac{29}{4}}} \end{matrix} \Big\}$ Latus igitur quæſitũ eſt $\sqrt{b:\frac{7}{2} + \sqrt{\frac{29}{4}}}$ pl $\sqrt{r:\frac{7}{2} - \sqrt{\frac{29}{4}}}$: & dicitur Major.

Exempl: V. Quæratur latus Binomii V, $\sqrt{20} + 4$; nempe $Z + 2Æ$. Quare $\frac{1}{2}\sqrt{}$ eſt $\sqrt{5}$: & $Æ$ eſt 2: & $\frac{1}{4}Zq - Æq$ eſt $5 - 4$; hoc eſt 1, cujus latus 1 eſt $\frac{1}{2}X$. At per Reg. $\sqrt{5} \pm 1 = \begin{matrix} \sqrt{5} + 1. & \sqrt{b:\sqrt{5} + 1} \\ \sqrt{5} - 1. & \sqrt{r:\sqrt{5} - 1} \end{matrix} \Big\}$ Latus igi-

tur

tur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+1$:pl $\sqrt{}$r:$\sqrt{5}-1$. Et dicitur Potens rationale cum mediali.

Exempl: VI. Quæratur latus Binomii VI, $\sqrt{20}+\sqrt{8}$: nempe Z+2Æ. Quare ; Z eſt $\sqrt{5}$: & Æ eſt $\sqrt{2}$: Et ; Zq -Æq eſt $5-2$; hoc eſt 3: cujus latus $\sqrt{3}$ eſt ; X. At per Regul: $\sqrt{5}\pm\sqrt{3}=$ $\left.\begin{matrix}\sqrt{5}+\sqrt{3}.\sqrt{}b:\sqrt{5}+\sqrt{3}.\\ \sqrt{5}-\sqrt{3}.\sqrt{}r:\sqrt{5}-\sqrt{3}.\end{matrix}\right\}$ Latus igitur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+\sqrt{3}$:pl$\sqrt{}$r:$\sqrt{5}-\sqrt{3}$. Et dicitur Potens duo medialia.

12 Atque hîc obiter trianguli rectanguli plani Geneſis ſe offert. Quia Zq = Xq+4AE. nempe Hq = Bq+Cq, per 47 è 1: Propoſitis binis quibuſcunque lineis ſive numeris A & E, trianguli rectanguli latera erunt, A+E, A-E, $\sqrt{}$4AE: vel etiam (mutatis A & E in Aq & Eq) Aq+Eq.Aq-Eq, 2AE. Ut ſi proponantur duo numeri 2 & 1: latera erunt 3,1,$\sqrt{8}$: nempe 2+1, 2-1, $\sqrt{4\times2\times1}$. vel etiam 5,3,4: nempe 4+1,4-1,2×1 bis.

13 Datis binis triangulis, rectangulis, H, B, C: & h, b, c: tertium ex ipſis fabricare: idque dupliciter.

1°. Quia $\left.\begin{matrix}\text{Bq}=\text{Hq}-\text{Cq}\\ \text{Et bq}=\text{hq}-\text{cq}\end{matrix}\right\}$ Multiplicentur invicem;

Eritque Bqbq = Hqhq+Cqcq miHqcq+Cqhq.
At Hqhq+Cqcq+2HChc = Q: Hh+Cc:
Et Hqcq+Cqhq+2HChc = Q. Hc+Ch:
Subducatur unum quadratum ex altero: & erit,
Bqbq = Q:Hh+Cc: mi Q:Hc+Ch:
Et ſic inventum eſt ex his triangulum tertium,

Bb. Hh+Cc.Hc+Ch. Hæc Regula ſit I.
Enunciatur autem verbis ſic. Pro trianguli novi baſe, ſumatur rectangulum ſub baſibus. Pro hypotenuſa, rectangulum ſub hypotenuſis, auctum rectangulo ſub cathetis. Pro catheto, rectangulum ſub hypotenuſa primi & catheto ſecundi, auctum rectangulo ſub catheto primi & hypotenuſa ſecundi.

II° Quia Hq=Bq+Cq } Multiplicentur invicem
Et hq=bq+cq }

Eritque Hqhq=Bqbq+Cqcq pl Bqcq+Cqbq.
At Bqbq+Cqcq−2BCBc=Q:Bb−Cc:
Et Bqcq+Cqbq+2BCbc=Q:Bc+Cb:
Addantur hæc duo quadrata: & erit
Hqhq=Q:Bb−Cc:pl Q:Bc+Cb.
Et ſic inventum eſt ex his triangulum tertium,
Hh.Bb−Cc.Bc+Cb. Hæc Regula ſit II.

Enuntiatur autem verbis ſic. Pro trianguli novi hypotenuſa, ſumatur rectangulum ſub hypotenuſis. Pro baſe, rectangulum ſub baſibus minutum rectangulo ſub cathetis. Pro catheto, rectangulum ſub baſe primi, & catheto ſecundi, auctum rectangulo ſub catheto primi & baſe ſecundi.

14. Si trianguli rectanguli latera continuè multiplicentur juxta binas regulas modò inventas: Prima multiplicatio angulum baſi oppoſitum duplicabit: Secunda triplicabit: tertia quadruplicabit: & ſic ulterius.

Exempl:

tur quæſitum eſt $\surd b: \surd 5 + 1$:pl $\surd r: \surd 5 - 1$. Et dicitur Potens rationale cum mediali.

Exempl: VI. Quæratur latus Binomii VI, $\surd 20 + \surd 8$: nempe $Z + 2Æ$. Quare $\frac{1}{4}Z$ eſt $\surd 5$: & $Æ$ eſt $\surd 2$: Et $\frac{1}{4}Zq - Æq$ eſt $5-2$; hoc eſt 3: cujus latus $\surd 3$ eſt $\frac{1}{4}X$. At per Regul: $\surd 5 \pm \surd 3 = \begin{cases} \surd 5 + \surd 3 . \surd b: \surd 5 + \surd 3 . \\ \surd 5 - \surd 3 . \surd r: \surd 5 - \surd 3 . \end{cases}$ Latus igitur quæſitum eſt $\surd b: \surd 5 + \surd 3$:pl$\surd r: \surd 5 - \surd 3$. Et dicitur Potens duo medialia.

12 Atque hîc obiter trianguli rectanguli plani Geneſis ſe offert. Quia $Zq = Xq + 4AE$. nempe $Hq = Bq + Cq$, per 47 è 1: Propoſitis binis quibuſcunque lineis ſive numeris A & E, trianguli rectanguli latera erunt, $A+E$, $A-E$, $\surd 4AE$: vel etiam (mutatis A & E in Aq & Eq) $Aq + Eq . Aq - Eq$, $2AE$. Ut ſi proponantur duo numeri 2 & 1: latera erunt $3, 1, \surd 8$: nempe $2+1, 2-1, \surd 4 \times 2 \times 1$. vel etiam 5,3,4: nempe $4+1, 4-1, 2 \times 1$ bis.

13 Datis binis triangulis, rectangulis, H, B, C: & h, b, c: tertium ex ipſis fabricare: idque dupliciter.

1°. Quia $\left.\begin{array}{l} Bq = Hq - Cq \\ \text{Et } bq = hq - cq \end{array}\right\}$ Multiplicentur invicem;

Eritque $Bqbq = Hqhq + Cqcq$ mi $Hqcq + Cqhq$.

At $Hqhq + Cqcq + 2HChc = Q: Hh + Cc$:

Et $Hqcq + Cqhq + 2HChc = Q. Hc + Ch$:

Subducatur unum quadratum ex altero: & erit,

$Bqbq = Q: Hh + Cc$: mi $Q: Hc + Ch$:

Et ſic inventum eſt ex his triangulum tertium,

Bb.

Bb. Hh+Cc.Hc+Ch. Hæc Regula ſit I.
Enunciatur autem verbis ſic. Pro trianguli novi baſe, ſumatur rectangulum ſub baſibus. Pro hypotenuſa, rectangulum ſub hypotenuſis, auctum rectangulo ſub cathetis. Pro catheto, rectangulum ſub hypotenuſa primi & catheto ſecundi, auctum rectangulo ſub catheto primi & hypotenuſa ſecundi.

II° Quia Hq=Bq+Cq
Et hq=bq+cq } Multiplicentur invicem

Eritque Hqhq=Bqbq+Cqcq pl Bqcq+Cqbq.
At Bqbq+Cqcq−2BCBc=Q:Bb−Cc:
Et Bqcq+Cqbq+2BCbc=Q:Bc+Cb:
Addantur hæc duo quadrata: & erit
Hqhq=Q:Bb−Cc:pl Q:Bc+Cb.
Et ſic inventum eſt ex his triangulum tertium,
Hh.Bb−Cc.Bc+Cb. Hæc Regula ſit II.

Enuntiatur autem verbis ſic. Pro trianguli novi hypotenuſa, ſumatur rectangulum ſub hypotenuſis. Pro baſe, rectangulum ſub baſibus minutum rectangulo ſub cathetis. Pro catheto, rectangulum ſub baſe primi, & catheto ſecundi, auctum rectangulo ſub catheto primi & baſe ſecundi.

14. Si trianguli rectanguli latera continuè multiplicentur juxta binas regulas modò inventas: Prima multiplicatio angulum baſi oppoſitum duplicabit: Secunda triplicabit: tertia quadruplicabit: & ſic ulterius.

Exempl.

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tur quæſitum eſt $\sqrt{b}: \sqrt{5} + 1 : pl\ \sqrt{r}: \sqrt{5} - 1$. Et dicitur Potens rationale cum mediali.

Exempl: VI. Quæratur latus Binomii VI, $\sqrt{20} + \sqrt{8}$: nempe $Z + 2Æ$. Quare $\frac{1}{2}Z$ eſt $\sqrt{5}$: & $Æ$ eſt $\sqrt{2}$: Et $\frac{1}{4}Zq - Æq$ eſt $5 - 2$; hoc eſt 3: cujus latus $\sqrt{3}$ eſt $\frac{1}{2}X$. At per Regul: $\sqrt{5} \pm \sqrt{3} = \left.\begin{matrix} \sqrt{5}+\sqrt{3}.\sqrt{b}: \sqrt{5}+\sqrt{3}. \\ \sqrt{5}-\sqrt{3}.\sqrt{r}: \sqrt{5}-\sqrt{3}. \end{matrix}\right\}$ Latus igitur quæſitum eſt $\sqrt{b}: \sqrt{5}+\sqrt{3} : pl\ \sqrt{r}: \sqrt{5}-\sqrt{3}$. Et dicitur Potens duo medialia.

12 Atque hîc obiter trianguli rectanguli plani Geneſis ſe offert. Quia $Zq = Xq + 4AE$. nempe $Hq = Bq + Cq$, per 47 è 1: Propoſitis binis quibuſcunque lineis ſive numeris A & E, trianguli rectanguli latera erunt, $A+E$, $A-E$, $\sqrt{4AE}$: vel etiam (mutatis A & E in Aq & Eq) $Aq+Eq$. $Aq-Eq$, $2AE$. Ut ſi proponantur duo numeri 2 & 1: latera erunt 3, 1, $\sqrt{8}$: nempe $2+1$, $2-1$, $\sqrt{4 \times 2 \times 1}$. vel etiam 5, 3, 4: nempe $4+1$, $4-1$, 2×1 bis.

13 Datis binis triangulis, rectangulis, H, B, C: & h, b, c: tertium ex ipſis fabricare: idque dupliciter.

I°. Quia $\left.\begin{matrix} Bq = Hq - Cq \\ \text{Et } bq = hq - cq \end{matrix}\right\}$ Multiplicentur invicem:

Eritque $Bqbq = Hqhq + Cqcq$ mi $Hqcq + Cqhq$.

At $Hqhq + Cqcq + 2HChc = Q: Hh + Cc$:

Et $Hqcq + Cqhq + 2HChc = Q. Hc + Ch$:

Subducatur unum quadratum ex altero: & erit,

$Bqbq = Q: Hh + Cc$: mi $Q: Hc + Ch$:

Et ſic inventum eſt ex his triangulum tertium,

Bb.

Bb.Hh+Cc.Hc+Ch. Hæc Regula ſit I.

Enunciatur autem verbis ſic. Pro trianguli novi baſe, ſumatur rectangulum ſub baſibus. Pro hypotenuſa, rectangulum ſub hypotenuſis, auctum rectangulo ſub cathetis. Pro catheto, rectangulum ſub hypotenuſa primi & catheto ſecundi, auctum rectangulo ſub catheto primi & hypotenuſa ſecundi.

II° Quia Hq=Bq+Cq
Et hq=bq+cq } Multiplicentur invicem

Eritque Hqhq=Bqbq+Cqcq pl Bqcq+Cqbq.
At Bqbq+Cqcq−2BCBc=Q:Bb−Cc:
Et Bqcq+Cqbq+2BCbc=Q:Bc+Cb:
Addantur hæc duo quadrata: & erit
Hqhq=Q:Bb−Cc:pl Q:Bc+Cb.
Et ſic inventum eſt ex his triangulum tertium,

Hh.Bb−Cc.Bc+Cb. Hæc Regula ſit II.

Enuntiatur autem verbis ſic. Pro trianguli novi hypotenuſa, ſumatur rectangulum ſub hypotenuſis. Pro baſe, rectangulum ſub baſibus minutum rectangulo ſub cathetis. Pro catheto, rectangulum ſub baſe primi, & catheto ſecundi, auctum rectangulo ſub catheto primi & baſe ſecundi.

14. Si trianguli rectanguli latera continuè multiplicentur juxta binas regulas modò inventas: Prima multiplicatio angulum baſi oppoſitum duplicabit: Secunda triplicabit: tertia quadruplicabit: & ſic ulterius.

Exempl:

tur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+1$:pl $\sqrt{}$r:$\sqrt{5}-1$. Et dicitur Potens rationale cum mediali.

Exempl: VI. Quæratur latus Binomii VI, $\sqrt{20}+\sqrt{8}$: nempe Z+2Æ. Quare $\frac{1}{2}$Z eſt $\sqrt{5}$:& Æ eſt $\sqrt{2}$: Et $\frac{1}{4}$ Zq -Æq eſt 5−2; hoc eſt 3: cujus latus $\sqrt{3}$ eſt $\frac{1}{2}$X. At per Regul: $\sqrt{5}\pm\sqrt{3}=$ $\left.\begin{matrix}\sqrt{5}+\sqrt{3}.\sqrt{}b:\sqrt{5}+\sqrt{3}.\\ \sqrt{5}-\sqrt{3}.\sqrt{}r:\sqrt{5}-\sqrt{3}.\end{matrix}\right\}$ Latus igitur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+\sqrt{3}$:pl$\sqrt{}$r:$\sqrt{5}-\sqrt{3}$. Et dicitur Potens duo medialia.

12 Atque hîc obiter trianguli rectanguli plani Geneſis ſe offert. Quia Zq = Xq+4AE. nempe Hq = Bq+Cq, per 47 è 1: Propoſitis binis quibuſcunque lineis ſive numeris A & E, trianguli rectanguli latera erunt, A+E, A-E, $\sqrt{4AE}$: vel etiam (mutatis A & E in Aq & Eq) Aq+Eq. Aq-Eq, 2AE. Ut ſi proponantur duo numeri 2 & 1: latera erunt 3, 1, $\sqrt{8}$: nempe 2+1, 2−1, $\sqrt{4\times2\times1}$. vel etiam 5, 3, 4: nempe 4+1, 4−1, 2×1 bis.

13 Datis binis triangulis, rectangulis, H, B, C: & h, b, c: tertium ex ipſis fabricare: idque dupliciter.

1°. Quia $\left.\begin{matrix}Bq = Hq - Cq\\ Et\ bq = hq - cq\end{matrix}\right\}$ Multiplicentur invicem;

Eritque Bqbq = Hqhq+Cqcq mi Hqcq+Cqhq.

At Hqhq+Cqcq+2HChc = Q: Hh+Cc:

Et Hqcq+Cqhq+2HChc = Q. Hc+Ch:

Subducatur unum quadratum ex altero: & erit,

Bqbq = Q:Hh+Cc: mi Q:Hc + Ch:

Et ſic inventum eſt ex his triangulum tertium,

$Bb . Hh + Cc . Hc + Ch$. Hæc Régula ſit I.
Enunciatur autem verbis ſic. Pro trianguli novi baſe, ſumatur rectangulum ſub baſibus. Pro hypotenuſa, rectangulum ſub hypotenuſis, auctum rectangulo ſub cathetis. Pro catheto, rectangulum ſub hypotenuſa primi & catheto ſecundi, auctum rectangulo ſub catheto primi & hypotenuſa ſecundi.

II° Quia $Hq = Bq + Cq$
Et $hq = bq + cq$ } Multiplicentur invicem

Eritque $Hqhq = Bqbq + Cqcq$ pl $Bqcq + Cqbq$.
At $Bqbq + Cqcq - 2BCBc = Q : Bb - Cc$:
Et $Bqcq + Cqbq + 2BCbc = Q : Bc + Cb$:
Addantur hæc duo quadrata : & erit
$Hqhq = Q : Bb - Cc :$ pl $Q : Bc + Cb$.
Et ſic inventum eſt ex his triangulum tertium,
$Hh . Bb - Cc . Bc + Cb$. Hæc Regula ſit II.

Enuntiatur autem verbis ſic. Pro trianguli novi hypotenuſa, ſumatur rectangulum ſub hypotenuſis. Pro baſe, rectangulum ſub baſibus minutum rectangulo ſub cathetis. Pro catheto, rectangulum ſub baſe primi, & catheto ſecundi, auctum rectangulo ſub catheto primi & baſe ſecundi.

14. Si trianguli rectanguli latera continuè multiplicentur juxta binas regulas modò inventas : Prima multiplicatio angulum baſi oppoſitum duplicabit : Secunda triplicabit : tertia quadruplicabit : & ſic ulterius.

Exempl.

tur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+1$:pl $\sqrt{}$r:$\sqrt{5}-1$. Et dicitur Potens rationale cum mediali.

Exempl: VI. Quæratur latus Binomii VI, $\sqrt{20}+\sqrt{8}$: nempe Z+2Æ. Quare $\frac{1}{2}$Z eſt $\sqrt{5}$:& Æ eſt $\sqrt{2}$: Et $\frac{1}{4}$Zq -Æq eſt 5-2; hoc eſt 3: cujus latus $\sqrt{3}$ eſt $\frac{1}{2}$X. At per Regul: $\sqrt{5}\pm\sqrt{3}=\left.\begin{array}{l}\sqrt{5}+\sqrt{3}.\sqrt{}b:\sqrt{5}+\sqrt{3}.\\ \sqrt{5}-\sqrt{3}.\sqrt{}r:\sqrt{5}-\sqrt{3}.\end{array}\right\}$ Latus igitur quæſitum eſt $\sqrt{}$b: $\sqrt{5}+\sqrt{3}$:pl$\sqrt{}$r:$\sqrt{5}-\sqrt{3}$. Et dicitur Potens duo medialia.

12 Atque hìc obiter trianguli rectanguli plani Geneſis ſe offert. Quia Zq = Xq+4AE. nempe Hq = Bq+Cq, per 47 è 1: Propoſitis binis quibuſcunque lineis ſive numeris A & E, trianguli rectanguli latera erunt, A+E, A-E, $\sqrt{4AE}$: vel etiam (mutatis A & E in Aq & Eq) Aq+Eq. Aq-Eq, 2AE. Ut ſi proponantur duo numeri 2 & 1: latera erunt 3,1,$\sqrt{8}$: nempe 2+1, 2-1, $\sqrt{4\times2\times1}$. vel etiam 5,3,4: nempe 4+1,4-1,2×1 bis.

13 Datis binis triangulis, rectangulis, H, B, C: & h, b, c: tertium ex ipſis fabricare: idque dupliciter.

I°. $\left.\begin{array}{l}\text{Quia Bq}=\text{Hq-Cq}\\ \text{Et bq}=\text{hq-cq}\end{array}\right\}$ Multiplicentur invicem;

Eritque Bqbq = Hqhq+Cqcq mi Hqcq+Cqhq.

At Hqhq+Cqcq+2HChc = Q: Hh+Cc:

Et Hqcq+Cqhq+2HChc = Q. Hc+Ch:

Subducatur unum quadratum ex altero: & erit,

Bqbq = Q:Hh+Cc: mi Q:Hc + Ch:

Et ſic inventum eſt ex his triangulum tertium,

Bb.

Bb.Hh+Cc.Hc+Ch. Hæc Régula ſit I.

Enunciatur autem verbis ſic. Pro trianguli novi baſe, ſumatur rectangulum ſub baſibus. Pro hypotenuſa, rectangulum ſub hypotenuſis, auctum rectangulo ſub cathetis. Pro catheto, rectangulum ſub hypotenuſa primi & catheto ſecundi, auctum rectangulo ſub catheto primi & hypotenuſa ſecundi.

II° Quia Hq = Bq + Cq }
Et hq = bq + cq } Multiplicentur invicem

Eritque Hqhq = Bqbq + Cqcq pl Bqcq + Cqbq.
At Bqbq + Cqcq − 2BCBc = Q:Bb − Cc :
Et Bqcq + Cqbq + 2BCbc = Q:Bc + Cb :
Addantur hæc duo quadrata : & erit
Hqhq = Q:Bb − Cc:pl Q:Bc + Cb.
Et ſic inventum eſt ex his triangulum tertium,
Hh.Bb − Cc.Bc + Cb. Hæc Regula ſit II.

Enuntiatur autem verbis ſic. Pro trianguli novi hypotenuſa, ſumatur rectangulum ſub hypotenuſis. Pro baſe, rectangulum ſub baſibus minutum rectangulo ſub cathetis. Pro catheto, rectangulum ſub baſe primi, & catheto ſecundi, auctum rectangulo ſub catheto primi & baſe ſecundi.

14. Si trianguli rectanguli latera continuè multiplicentur juxta binas regulas modò inventas : Prima multiplicatio angulum baſi oppoſitum duplicabit : Secunda triplicabit : tertia quadruplicabit : & ſic ulterius.

Exempl.

Exempl: Reg: I^a. Bb. Hh+Cc.Hc.+Ch.

B. H. C: anguli ſimpli.

B. H. C

Bq . Hq+Cq . 2HC . anguli dupli.

B . H . C

Bc . Hc+HCq . 2HqC

2HCq . HqC+Cc.

Bc . Hc+3HCq . 3HqC+Cc. tripli.

B . H . C

Bqq . Hqq+3HqCq . 3HcC+HCc

Cqq+3HqCq HcC+3HCc

Bqq . Hqq+6HqCq+Cqq . 4HcC+4HCc.

B . H . C quadrupli.

&c

Exempl: Reg: II^a. Hh. Bb−Cc. Bc+Cb.

H . B . C : anguli ſimpli.

H B . C

Hq . Bq−Cq . 2BC : anguli dupli.

H . B . C

Hc . Bc−BCq . 2BqC

−2BCq BqC−Cc

Hc . Bc−3BCq . 3BqC−Cc : tripli.

H . B . C

Hqq . Bqq−3BqCq . 3BcC−BCc

Cqq−3BqCq BcC−3BCc

Hqq . Bqq−6BqCq+Cqq . 4BcC−4BCc.

H . B . C quadrupli.

&c

Cap. XVII.

Alia tabulæ poſterioris in Cap: 12. inſpectio, quoàd Æquationes.

1 A Binomia radice A + E, poteſtatum ſpecies omnes ſunt adfirmatæ. A Reſiduo verò poteſtatum ſpecies omnes ſunt alternatim negatæ, ut Q: A − E : eſt Aq -- 2 A E + Eq. Et C: A − E : eſt Ac − 3 AqE + 3 AEq - Ec. Et QQ: A - E : eſt Aqq - 4AcE + 6AqEq − 4AEc + Eqq. &c Adeò ut ſi poteſtatis cujuſvis ſpecies alternatim ſumptæ, in duas ſummas aggregentur; harum ſummarum connexio cum ſigno radicis, erit radicis ipſius poteſtas. Atque hæc eſt Binomiorum, ac Reſiduorum, Quadraticorum, Cubicorum, aliorumque conſtitutio.

2. Quare nominum Binomii, vel Reſidui cujuſque differentia, eſt homogenea poteſtas differentiæ nominum radicis. ſcil: Ac + 3 AEq mi 3 AqE + Ec, vel Ac + 3 AEq − 3 AqE - Ec, eſt C : A - E.

3. Et Quadratorum è nominibus Binomii vel Reſidui cujuſque differentia, eſt homogenea poteſtas differentiæ quadratorum è nominibus radicis. ſcil: Q: Ac + 3 AEq : mi Q: 3 AqE + Ec : eſt C : Aq - Eq.

Nam per exempl: Reg: I, in 14, Cap: XVI, ſi cogitetur A hypotenuſa trianguli rectanguli; & E cathetus; & Aq − Eq quadratum baſis; hoc Theorema aliter ſymbolis explicabitur ſic : Q: Hc + 3HCq: mi Q: 3HqC + Cc: = C: Bq: = Q: Bc: ergo

4 At ſi ſpecies in nominibus aggregatæ, ipſæ etiam alternatim adfirmentur, & negentur: Quadratorum è nominibus ſumma, eſt homogenea poteſtas ſummæ quadratorum è nominibus radicis. Scil: Q: Ac−3AEq: pl Q: 3AqE−Ec: eſt C: Aq+Eq.

Nam per exempl: Reg:II, in 14, Cap:XVI, ſi cogitetur A baſis trianguli rectanguli; & E cathetus; & Aq+Eq quadratum hypotenuſæ; hoc Theorema aliter ſymbolis explicabitur ſic: Q: Bc−3BCq:pl Q: 3BqC−Cc:=C:Hq:=Q:Hc. Ergo

Pulcherrima ſunt hæc circa angulares ſectiones myſteria.

5 Omnes cujuſque ordinis intermediæ ſpecies, ſunt etiam poteſtates mediorum inter A & E proportionalium. ſcil. inter Ac & Ec, ſunt duæ mediæ proportionales, AqE & AEq: qui etiam cubi ſunt ex M & N. Quare A, √cAqE, √cAEq, E, ſunt continuè proportionales: nempe A,M,N,E. Nam AqE=AMN=Mc: & AEq=MNE=Nc. Atque hinc patet inventio quotlibet mediorum proportionalium inter A & E: ut ſi velis quinque medios proportionales, poteſtates erunt [6] ſive cc, quarum Index unitate excedit numerum quæſitorum mediorum: Eruntque A, √cc AqcE, √ccAqqEq, √cc Ac Ec, √cc Aq Eqq, √cc AEqc, E, ∺.

6 Omnis media ſpecies in unoquoque genere, fit ex duabus nominum radicis poteſtatibus, quarum Indices ſimul, æquales ſunt Indici ejuſdem generis: mediæ

diæ autem ipſius ſpeciei ab extremis ſuis diſtantiæ, æquales erunt Indicibus alternarum facientium: & facientibus ſuis in communi angulo reſpondent. Scil: AqEc generis quadrato-cubici, fit ex Aq in Ec, quibus in communi angulo reſpondent. Eſtque ab Aqc tertia: & ab Eqc ſecunda.

Conſect. Atque hinc facile erit radicis Binomiæ datæ poteſtatem quamlibet (inventis omnibus mediis inter poteſtates nominum extremas) conſtruere. Ut in exemplo, ſi radicis Binomiæ A + √ Æ quæratur Quadrato-Cubus: erit Aqc + 10 ÆAc + 5 ÆqA plus 5 √ÆAqcc + 10√Æc Aqq + √Æqc: Quod Binomium eſt Quadrato-Cubic.

A Aq Ac Aqq Aqc
√Æ Æ, √Æc Æq √Æqc

5√ÆAqcc
10 ÆAc
10√Æc Aqq
5 ÆqA
√Æqc

7. Si ſpecies aliqua multiplicetur in Æ, producta magnitudo erit media ſpecies collateralis, in ordine alternè ſequente, atque eadem numero à ſuis extremis. Ut Ac × Æ, eſt AqqE, quæ prima eſt ab Aqc, & quarta ab Eqc. Sic AcE × Æ, eſt Aqq Eq, quæ ab Acc ſecunda eſt, & ab Ecc quarta. Et ſimiliter de reliquis.

8 Si ſpecies aliqua multiplicetur per A — E vel X, producta magnitudo erit differentia inter duas ſpecies ordinis ſequentis utrinque proximas. Ut AcX = Aqq — AcE. AqEX = AcE — AqEq. AEqX = AqEq — AEc. EcX = AEc — Eqq. Quare

Si

Si omnes cujuſvis ordinis ſpecies multiplicentur per X, producetur differentia duarum poteſtatum extremarum ordinis proximi ſuperioris. Ut ex Ac+AqE+AEq+Ec, ductis in X, fiet Aqq-Eqq.

9 In ordinibus Indicum imparium (l, c, qc, &c) ſumma duarum extremarum poteſtatum; at in ordinibus Indicum parium (q, qq, cc, &c) differentia earundem; fit ex A+E ducta in ſingulas ſpecies ordinis minoris præcedentis, alternatim adfirmatas & negatas. Ut Ac+Ec, fit ex Aq—AE+Eq, ductis in A+E. Item Aqq-Eqq, fit ex Ac-AqE+AEq-Ec, ductis in A+E.

10 Si eadem magnitudo multiplicetur in duas magnitudines contrarias: magnitudines ex ipſis factæ erunt etiam contrariæ. Ut Aq-2AE+Eq ductæ in A-E, fient Ac--3AqE+3AEq--Ec. At verò eædem ductæ in --A+E, fient--Ac+3AqE--3AEq+Ec.

11 Unciæ ſive numeri ſpeciebus præfixi, ſunt figuræ numerariæ. Nam omnes ſub A & E, ſunt radices. Omnes ſub Aq & Eq, ſunt triangulares. Omnes ſub Ac & Ec, ſunt pyramidales. Omnes ſub Aqq & Eqq, ſunt triangulo-triangulares. Omnes ſub Aqc & Eqc, ſunt triangulo-pyramidales. Omnes ſub Acc & Ecc, ſunt pyramidi-pyramidales, &c.

12. Si radix tribus conſtet nominibus, A, E, I,
Quadratum: Aq, 2AE, Eq, 2EI, Iq, 2AI,
Cubus: Ac, Ec, Ic, 3AqE, 3AEq, 3AqI, 3AIq, 3EqI, 3EIq, 6AEI.

Et nota quòd ſi in aliqua ſpecie, numerus laterum negatorum ſit impar; ſpecies illa erit negata. Ut Q: A+E−I: = Aq+2AE+Eq−2EI+Iq−2AI. Et C: A+E−I: = Ac+3AqE+3AEq+Ec−3EqI+3EIq−Ic−3AqI+3AIq−6AEI.

Cap. XVIII.
Penus Analytica.

1 EX primis ac facillimis æquationibus, quæ nihil aliud ſunt, quàm vel terminorum expoſitiones, vel ſimplices effectiones (quales ſunt illæ capitis XI, $\frac{1}{2}Z - E = \frac{1}{2}X$: & $\frac{1}{2}X + E = \frac{1}{2}Z$: & reliquæ cujuſmodi) innumeræ aliæ deducuntur, per Additionem, Subductionem, multiplicationem, Diviſionem, Tranſpoſitionem, atque Interpretationem: ſumendo id quod alteri inventum eſt æquale, loco ejus cui æquatur. Quæ quidem Analytica ſupellex eſt, non minùs pretioſa, quàm copioſa. Quarum ego præcipuas aliquot, & maximè neceſſarias adſcribam: plures Analytices ſtudioſus pro ſuo exercitio excogitabit. Et ubicunque

ſive in Arithmetica, ſive in Geometria, ſive in alia aliqua arte, incideret in magnitudinem aliquam, cui altera æqualis eſſe intelligitur; æqualitatem illam quibuſcunque poterit modis atque comparationibus, torquebit, diſcutiet, variabit, ut novum inde artis inſtrumentum inveniat: quod poſtea in penu ſervabit: & ubicunque poterit in uſum proferet, ad artis ſubſidium atque augmentum.

2|. $Q:1:=9Q:\frac{1}{3}$. &c.	$C:1:=27\,C:\frac{1}{3}:$ &c
$Q:1:=\frac{1}{9}Q:3$. &c	$C:1:=\frac{1}{27}\,C:3:$ &c
$Q:1:=\frac{9}{4}Q:\frac{2}{3}$. &c	$C:1:=\frac{27}{8}C:\frac{2}{3}:$ &c
$\frac{2}{3}Q:1:=\frac{2}{3}\times\frac{9}{4}Q:\frac{2}{3}:$ &c	$\frac{2}{3}C:1:=\frac{2}{3}\times\frac{27}{8}C:$ &c
$\frac{2}{3}Q:4:=\frac{2\times 16}{3}Q:1$. &c	$\frac{2}{3}C:4:=\frac{2\times 64}{3}C:1:$ &c

3 Si linea biſecetur, & ſecùs; rectangulum ſub ſegmentis inæqualibus, æquatur differentiæ quadratorum biſegmenti atque interſegmenti: hoc eſt ſemiſummæ atque ſemidifferentiæ ſegmentorum. 5 e 2. $AE=Q:\frac{1}{2}A+\frac{1}{2}E:$ mi $Q:\frac{1}{2}A-\frac{1}{2}E:$ hoc eſt, $AE=\frac{1}{4}Zq-\frac{1}{4}Xq$.

4 Si linea biſecta augeatur; rectangulum ſub tota aucta & augmento, æquatur differentiæ quadratorum biſegmenti aucti, atque biſegmenti. 6 e 2. $A+E$ in $E=Q:\frac{1}{2}A+E:$ mi $Q:\frac{1}{2}A$. Et $A+E$ in $A=Q:\frac{1}{2}E+A:$ mi $Q:\frac{1}{2}E$.

Datis igitur ſumma trium ∺ $(Aq+AE+Eq)$ cum alterutro extremorum, dantur duo reliqui. Nam.

$\sqrt{u}: Aq+AE+Eq-\frac{3}{4}Aq:$ mi $\frac{1}{2}A=E$.

$\sqrt{u}: Aq+AE+Eq-\frac{3}{4}Eq:$ mi $\frac{1}{2}E=A$.

5 Si

5 Si linea ſecetur utcunque; ſumma quadratorum totius, & unius ſegmenti, æquatur aggregato quadrati alterius ſegmenti, & duplicis rectanguli ſub tota & priore ſegmento, 7 e 2. $Zq+Aq=2ZA+Eq$. Et $Zq+Eq=2ZE+Aq$. Quare $2ZA+Eq-Aq=Zq=2ZE+Aq-Eq$.

6 Si linea utcunque ſecta, augeatur alterutro ſegmento; Quadruplex rectangulum ſub ſecta, & ſegmento augente, æquatur differentiæ quadratorum totius auctæ, & alterius ſegmenti. 8 e 2.

$Q{:}Z+E{:}-Aq=4ZE$. Et $Q{:}Z+A{:}-Eq=4ZA$.

7 Si linea biſecetur, & ſecùs; ſumma quadratorum ſegmentorum inæqualium, æquatur duplicatæ ſummæ quadratorum biſegmenti, & interſegmenti. 9 e 2. $Aq+Eq=2Q{:}\frac{1}{2}A+\frac{1}{2}E{:}+2Q{:}\frac{1}{2}A-\frac{1}{2}E$.

8 Si linea biſecta augeatur; ſumma quadratorum totius auctæ & augmenti, æquatur duplicatæ ſummæ quadratorum biſegmenti aucti, & biſegmenti. 10 e 3.

$Q{:}A+E{:}+Eq=2Q{:}\frac{1}{2}A+E{:}+2Q{:}\frac{1}{2}A$.

$Q{:}A+E{:}+Aq=2Q{:}\frac{1}{2}E+A{:}+2Q{:}\frac{1}{2}E$.

9 $Aq=ZA-AE=XA+AE=\frac{1}{2}ZA+\frac{1}{2}XA=Q{:}Z-E{:}=Q{:}E+X{:}=Z-Eq=Eq+X$.
Et $Eq=ZE-AE=AE-XE=\frac{1}{2}ZE-\frac{1}{2}XE=Q{:}Z-A{:}=Q{:}A-X{:}=Z-Aq=Aq-X$.

10 $Æ=\frac{1}{4}Zq-\frac{1}{4}Xq=ZA-Aq=ZE-Eq=Aq-XA=Eq+XE=\frac{1}{2}Zq-\frac{1}{2}Z=\frac{1}{2}Z-\frac{1}{2}Xq=\frac{1}{2}ZA-\frac{1}{2}XA=\frac{1}{2}ZE+\frac{1}{2}XE$.

11. $\mathcal{Z} = Aq+Eq = Zq - 2AE = 2AE+Xq = ZE+XA = ZA-XE = 2Q{:}\tfrac{1}{2}Z{:}+2Q{:}\tfrac{1}{2}Z-E{:} = Q{:}A-2N{:}+Q{:}2M-E{:} = \tfrac{1}{2}Zq+\tfrac{1}{2}Xq = 2Q{:}\tfrac{1}{2}Z{:}+2Q{:}\tfrac{1}{2}X{:}$ Conſectarium ex his duabus ultimis æquationibus: Si magnitudó conſtet ex quadratis binarum magnitudinum: ejus etiam duplum conſtabit ex duobus quadratis, ſummæ ſcil: & Differentiæ. Et dimidium ejus conſtabit ex duobus quadratis, ſemiſummæ ſcil: & Semidifferentiæ.

Et $\mathcal{X} = Aq-Eq = ZX = 2XA-Xq = 2XE+Xq = ZA-ZE = XA+XE = Zq-2ZE = ZA+XE-2Æ = XA+2Æ-ZE = Q{:}A+2N{:}$ mi $Q{:}2M+E.$

12. $Q{:}\tfrac{1}{2}A+\tfrac{1}{2}E{:} = Q{:}\tfrac{1}{2}A-\tfrac{1}{2}E{:}+Æ.$

Et $Q{:}\tfrac{1}{2}A-\tfrac{1}{2}E{:} = Q{:}\tfrac{1}{2}A+\tfrac{1}{2}E{:}-Æ.$

13. $2A+2E$ in $A = 2Aq+2AE = Zq+\mathcal{X}.$

Et $2A-2E$ in $A = 2Aq-2AE = \mathcal{X}+Xq.$

Et $2A+2E$ in $E = 2AE+2Eq = Zq-\mathcal{X}.$

Et $2A-2E$ in $E = 2AE-2Eq = \mathcal{X}-Xq.$

14. $\mathcal{X}q = ZqXq = \mathcal{Z}+2Æ$ in $\mathcal{Z}-2Æ = \mathcal{Z}q - 4AqEq.$

15. $ZÆ = AqE+AEq.$ Et $XÆ = AqE-AEq.$

Et $\mathcal{Z}Æ = AcE+AEc.$ Et $\mathcal{X}Æ = AcE-AEc.$

Quare $\mathfrak{z}+3ZÆ = Zc.$ Et $\mathfrak{x}-3XÆ = Xc.$

Et $\mathcal{Z}Z = \mathfrak{z}+ZÆ = Ac+AqE+AEq+Ec.$

Et $\mathcal{Z}X = \mathfrak{x}-XÆ = Ac-AqE+AEq-Ec.$

Et $\mathcal{X}Z = \mathfrak{x}+XÆ = Ac+AqE-AEq-Ec.$

Et $\mathcal{X}\mathfrak{x} = \mathcal{Z}-ZÆ = Ac-AqE-AEq+Ec.$

Hinc $\mathcal{Z}Z+\mathcal{X}X = 2\mathfrak{z}.$ Et $\mathcal{X}Z+\mathcal{Z}X = 2\mathfrak{x}.$

Et $\mathcal{Z}Z-\mathcal{X}X = 2ZÆ.$ Et $\mathcal{X}Z-\mathcal{Z}X = 2XÆ.$

16 Si

16. Si in circulo ſit $7.22::\delta.\pi::113.355$: erit $\delta.\pi::$ 2R.P: &

$\delta.\pi.:Rq$. Circul. Et $\pi.\ \delta::\frac{1}{4}Pq$. Circul.

$\delta.\pi::2Rc$. Cylind. Et $\pi q.\delta q::\frac{1}{4}Pc$. Cylind.

$\delta.\pi::\frac{2}{3}Rc$. Sphær. Et $\pi q.\delta q::\frac{1}{6}Pc$. Sphær.

$\delta.\pi::\frac{1}{3}Rc$. Con. Et $\pi q.\delta q::\frac{1}{12}$, Pc. Con.

17. Ad hæc oportet futuram Analyſtam Geometrica iſta, tum theoremata, tum problemata non ignorare.

Theor: 1. Triangula ſunt æqualia: Si in utroque, vel tria latera; vel duo latera cum angulo comprehenſo; vel duo latera cum angulo eidem lateri oppoſito; vel duo anguli cum latere interjacente; vel duo anguli cum latere eidem ſubtenſo; æquentur. 4, 8, 26, e 1.

Theor: 2. Triangula plana ſunt ſimilia: Si vel ſint æquiangula; vel lateribus omnibus proportionalia; vel habeant unum angulum æqualem, & alterum angulum crurum proportionalium, & angulum tertium homogeneum. 4, 5, 6, 7, e 6.

Theor: 3. In omni triangulo, majus latus majorem angulum ſubtendit; & minus minorem; & æquale æqualem. 18, 19 e 1.

Theor: 4. Duæ rectæ lineæ ſunt parallelæ: Si recta ipſas ſecans æquales fecerit, vel angulos alternos; vel externum & internum oppoſitum; vel duos internos ex eâdem parte duobus rectis. Et contra. 27, 28, 29, 30, e 1. Nam lineæ rectæ parallelæ ſunt inſtar unius lineæ latæ.

Theor: 5. Trianguli tres anguli ſimul, æquantur duobus rectis: Et externus angulus duobus internis oppoſitis. 32 e 1.

Theor: 6. Si rectam in circulo inſcriptam, recta è centro biſecet: ad angulos rectos ipſam ſecat. 3 e 3.

Theor: 7. Perpendicularis ſuper finem diametri, circulum tangit. 16, 18, 19, e 3.

Theor: 8. Angulus ad centrum duplus eſt anguli ad peripheriam. 20 e 3.

Theor: 9. In eodem, vel æqualibus circulis, anguli ſuper equalibus peripheriis, ſunt æquales. 21 e 3.

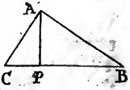
Theor: 10. Angulus in ſemicirculo eſt rectus. 31 e 3.

Theor: 11. Si è puncto in peripheria circuli ducantur binæ rectæ lineæ, una circulum tangens, altera ſecans: anguli inter ipſas comprehenſi menſura, æqualis erit ſemiperipheriæ abſciſſæ. pro: 32 e 3.

Theor: 12. Triangula, ſive parallelogramma, æquialta, vel inter eaſdem parallelas, ſunt ut baſes. 35, 36, 37, 38, e 1. & 1 e 6.

Theor: 13. Recta biſecans angulum trianguli, ſecat baſem ratione crurum 3 e 6.

Theor: 14. Triangulum rectangulum quodvis notetur literis A B C: ſicut A ſit angulus rectus: & B A Baſis: & C A Cathetus: B C Hypotenuſa.



Theor: 15. In triangulo rectangulo plano, perpendicularis ex angulo

gulo recto in Hypotenuſam, dividit triangulum in duo triangula, tum toti, tum ſibi ipſis ſimilia. 8 e 6.

BC. BA. CA :: BA. BP. AP :: CA. AP. CP.
Hypotenuſæ Baſes Catheti ;

Unde ſequitur.

1° Perpendicularem eſſe mediam proportionalem inter ſegmenta Hypotenuſæ. Ideoque Quadratum perpendicularis æquale eſſe rectangulo ſub ſegmentis. Scil: ∺ BP, AP, CP. Et APq = BP × CP.

2° Baſem eſſe mediam proportionalem inter Hypotenuſam, & ſegmentum Hypotenuſæ Baſi conterminum. Scil: ∺ BC, BA, BP.

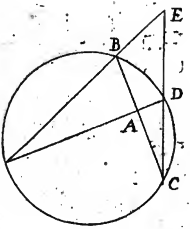
3° Cathetum eſſe mediam proportionalem inter Hypotenuſam, & ſegmentum Hypotenuſæ Catheto conterminum. Scil: ∺ BC, CA, CP.

4° Baſis & Catheti quadrata, eſſe ut ſegmenta Hypotenuſæ contermina. BP. CP :: BAq. CAq.

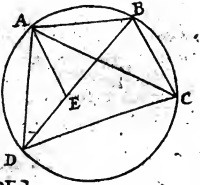
5° Quadratum Hypotenuſæ æquari quadratis Baſis & Catheti ſimul. BCq = BAq + CAq.

Theor: 16. Si in circulo duæ rectæ inſcriptæ ſeſe mutuò interſecent intra circulum (in puncto A); rectangulum ſub ſegmentis unius, æquale eſt rectangulo ſub ſegmentis alterius. 35 e 3.

Si verò ſeſe extra circulum interſecent (in puncto E) Rectangula ſub ſegmentis utriuſque à puncto ad convexum & concavum circuli, ſunt æqualia. 36 & 37 e 3. Dico primò AB×AC=AD ×AF. Nam tri: BAF, DAC ſim. Dico ſecundò EB×EF =ED×EC. Nam tri: BEC, DEF ſim.

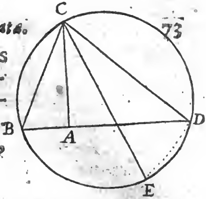


Theo: 17. Quadrilateri in circulo inſcripti anguli interiores oppoſiti ſimul æquantur duobus rectis. 22 e 3. Et ſi ducantur duo diagonii, rectangulum ſub diagoniis, æquale erit duobus rectangulis ſub lateribus oppoſitis, Dico AC×BD =AB×CD+AD×BC. Nam tri: ACB, ADE ſim. & ADC, AEB ſim.



Quare $\left\{ \begin{array}{l} AC.\ CB :: AD.DE \\ AC.CD :: AB.BE \end{array} \right\}$ ergo,

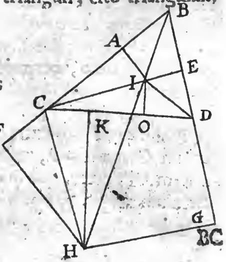
Theor: 18. Si ex angulo quovìs trianguli circulo inſcripti, demittatur perpendicularis in latus oppoſitum:



tum : Erit ut perpendicularis illa, ad unum crus ejuſdem anguli : ſic crus alterum, ad diametrum circuli. Dico CA. CB::CD.CE. Nam tri : ACB, DCE ſim.

Theor: 19. Triangula unum angulum æqualem habentia rationem habent eam, quæ ex lateribus componitur. 23 e 6.

Theor: 20. Si ſemiſumma trium laterum trianguli plani, & tres differentiæ trium laterum ab illa ſemiſumma, continuè inter ſe multiplicentur : Vel aliter, ſi trianguli quovis latere ſumpto pro baſe, & reliquis duobus pro cruribus ; Rectangulum ſub ſemiſumma & ſemidifferentia ſummæ crurum & baſis, ducatur in rectangulum ſub ſemiſumma & ſemidifferentia baſis & differentiæ crurum : Facti latus quadratum æquale erit areæ trianguli, eſto triangulum BCD, cujus crura ſint BC & BD, & baſis CD. Biſecentur tres anguli rectis BI, CI, DI, concurrentibus in I: unde in latera ad angulos rectos ducantur IA, IE, IO. Sunt igitur intra triangulum BCD, tria paria triangulorum æqualium. Quare ſi cruri



BC adjungatur in directum CF=DE; erit BF= $\frac{1}{2}$BC+$\frac{1}{2}$BD+$\frac{1}{2}$CD : Et BA=BF-CD=$\frac{1}{2}$BC+$\frac{1}{2}$BD -$\frac{1}{2}$CD: Et AC=BF-BD=$\frac{1}{2}$CD+$\frac{1}{2}$BC-$\frac{1}{2}$BD : Et CF =BF-BC=$\frac{1}{2}$CD-$\frac{1}{2}$BC+$\frac{1}{2}$BD. Menſuratis BG=BF: & CK=CF: ducantur perpendiculares FH,GH,KH : Et protrahatur BI in H. Quia ang: FCK+FHK= 2 Rect=FCK+ACO : Et ang:ACO+AIO=2Rect: Erunt quadrangula FCKH, AIOC ſim. Et tri: CFH, IAC ſim. Sunt etiam tri: BAI, BFH ſim. His expoſitis, Dico Quadratum arræ trianguli, nempe BFq×IAq=BF×BA×AC×CF.

Nam IA.BA::FH.BF }
Et IA.AC::CF.FH } propter tri: ſim.

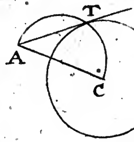
Quare per multipl:IAq×BF=BA×AC×CF.

Ducatur utraque pars in BF, eritque &c.

Probl: 1. A dato puncto, vel ad datam diſtantiam, datæ rectæ lineæ parallelam ducere. 31 e 1.

Probl: 2. Data recta linea, à dato in ea puncto, rectam lineam perpendicularem, ſive ad angulos rectos, excitare. 11 e 1.

Probl: 3. Super datam rectam lineam, à dato extra ipſam puncto, perpendicularem rectam demittere. 12 e 1.

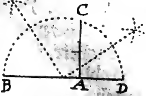


Probl. 4. A dato extra circulum C, puncto A, rectam lineam AT ducere, quæ ipſum circulum tangat. 17 e 3.

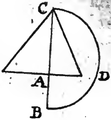
Probl:

Probl: 5. Tribus rectis lineis datis, quartam proportionalem adinvenire. 12 e 6.

Probl:6. Datis duabus rectis lineis AB, AD, mediam continuè proportionalem AC, adinvenire. 13 e 6.



Probl: 7. Datis duabus rectis lineis AB, AC, vel AD, AC, tertiam continuè proportionalem AD, vel AB, adinvenire. 11 e 6.

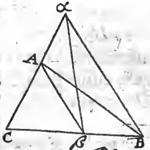


Probl:8. Dato triangulo, cujus altitudo eſt AC, & ſemibaſis AB, æquale quadratum ADq, conſtituere.

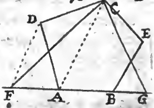
Probl:9. Dato rectangulo aliud rectangulum æquale, ad datum latus, ſtatuere. 14 e 6.

Probl: 10. Triangulo dato aliud triangulum æquale, ad datam altitudinem conſtituere.

Ex punctis altitudinum A & *α*, in angulos oppoſitos linea A*β* & *α*B, ductæ, ſint parallelæ.



Probl: 11. Dato polygono æquale triangulum conſtituere.



Probl: 12. Datis tribus punctis, non in directum poſitis, ducere circumferentiam. 25 e 3.

Probl:13.

Probl: 13. Datis trianguli rectanguli baſe & Catheto, invenire hypotenuſam; vel quadratum quadrato addere.

Probl: 14. Datis trianguli rectanguli hypotenuſa & baſe, invenire cathetum: vel quadratum ex quadrato tollere.

Probl: 15. Binarum figurarum ſimilium rationem invenire. Quæratur tertia proportionalis.

Probl: 16. Datæ figuræ ſimilem figuram, in data ratione conſtituere. Quæratur media proportionalis inter latus ipſius; & latus ſimile.

Probl: 17. In dato circulo hexagonum ordinatum inſcribere. 15 e 4.

Probl: 18. In dato circulo Decagonum ordinatum inſcribere. Secetur ſemidiameter circuli ſecundum extremam & mediam rationem, per 11 e 2.

Probl: 19. In dato circulo Pentagonum ordinatum inſcribere. Quæratur Hypotenuſa trianguli rectanguli, cujus Baſis ſit latus Hexagoni, & Cathetus latus Decagoni.

CAP. XIX.

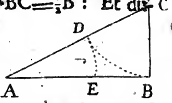
Exempla Æquationis Analyticæ, pro Theorematibus inveniendis, Problematibuſque ſolvendis. ad quem quaſi ſcopum præcepta hactenus tradita præcipuè collineantur.

Probl: I. Inventio 11 e 2. Nempe, Data recta linea B ſecetur ſic ut rectangulum ſub tota B, & minore ſegmento, æquetur quadrato majoris ſegmenti.

Po-

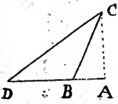
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Denique menſuretur $AE=AD$, pro majore ſegmento.

Probl: II. Inventio 12 e 2. Nempe comparatiò Baſis obtuſi anguli, cum lateribus. Eſto triangulum BCD : cujus angulus interior ad B, ſit obtuſus : hujus Baſis eſt DC: & latera BD, BC. Hîc $BCq-BAq=CAq=DCq$ ($-DAq$, per 4 e 2) $-BDq-2BD\times BA-BAq$. Quare $BCq+BDq=DCq-2BD\times BA$.



Quod theorema verbis enuntiatur, ſic : In amblygoniis triangulis, quadratum lateris ſubtendentis obtuſum angulum, excedit ſummam quadratorum laterum eundem comprehendentium, duplice rectangulo

ſub

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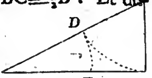
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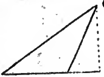
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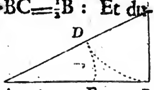
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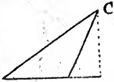
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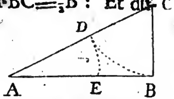
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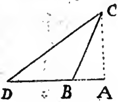
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Geometricè autem conſtruetur, ſic, Fiat AB=B : eique ad angulos rectos ſtatuatur BC=½B : Et ducatur Hypotenuſa AC : erit AC=√u: Bq+¼Bq. Abſcindatur CD=BC : Eritque reſiduum AD=√u: Bq+¼Bq: -½B. Denique menſuretur AE=AD, pro majore ſegmento.



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Probl: 16. Datæ figuræ ſimilem figuram, in data ratione conſtituere. Quæratur media proportionalis inter latus ipſius, & latus ſimile.

Probl: 17. In dato circulo hexagonum ordinatum inſcribere. 15 e 4.

Probl: 18. In dato circulo Decagonum ordinatum inſcribere. Secetur ſemidiameter circuli ſecundum extremam & mediam rationem, per 11 e 2.

Probl: 19. In dato circulo Pentagonum ordinatum inſcribere. Quæratur Hypotenuſa trianguli rectanguli, cujus Baſis ſit latus Hexagoni, & Cathetus latus Decagoni.

Cap. XIX.

Exempla Æquationis Analyticæ, pro Theorematibus inveniendis, Problematibuſque ſolvendis. ad quem quaſi ſcopum præcepta hactenus tradita præcipuè collineantur.

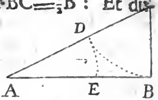
Probl: I. INventio 11 e 2. Nempe, Data recta linea B ſecetur ſic ut rectangulum ſub tota B, & minore ſegmento, æquetur quadrato majoris ſegmenti.

Po-

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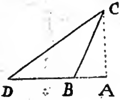
Ponatur majus ſegmentum A : minus erit $B-A$. ducatur $B-A$ in B: fietque $Bq-BA=Aq$: vel $Aq+BA=Bq$. Quare $\sqrt{u}$: $Bq+\frac{1}{4}Bq:-\frac{1}{2}B=A$, per 9 cap. 16. Quod Theorema verbis enunciatur ſic : Si quadrato lineæ datæ, addatur quadrati ipſius quadrans : & è latere quadrato ſummæ, tollatur ſemis lineæ datæ: reliquum erit ſegmentum majus.

Geometricè autem conſtruetur, ſic, Fiat $AB=B$: eique ad angulos rectos ſtatuatur $BC=\frac{1}{2}B$: Et ducatur Hypotenuſa AC : erit $AC=\sqrt{u}$: $Bq+\frac{1}{4}Bq$. Abſcindatur $CD=BC$: Eritque reſiduum $AD=\sqrt{u}$: $Bq+\frac{1}{4}Bq:-\frac{1}{2}B$.



Denique menſuretur $AE=AD$, pro majore ſegmento.

Probl: II. Inventio 12 e 2. Nempe comparatio Baſis obtuſi anguli, cum lateribus. Eſto triangulum BCD : cujus angulus interior ad B, ſit obtuſus: hujus Baſis eſt DC: & latera BD, BC. Hîc $BCq-BAq=CAq$ $=DCq$ ($-DAq$, per 4 e 2) $-BDq$ $-2BD\times BA-BAq$. Quare $BCq+BDq$ $=DCq-2BD\times BA$.

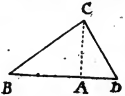


Quod theorema verbis enuntiatur, ſic: In amblygoniis triangulis, quadratum lateris ſubtendentis obtuſum angulum, excedit ſummam quadratorum laterum eundem comprehendentium, duplice rectangulo

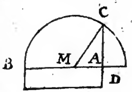
ſub

ſub uno laterum circa obtuſum angulum, & ſegmento ipſius inter obtuſum angulum & perpendiculum.

Probl: III. Inventio 13 e 2. Nempe comparatio Baſis acuti anguli, cum lateribus. Eſto triangulum BCD: cujus angulus interior ad B, ſit acutus. hujus Baſis eſt DC: & latera BC, BD. Hîc $BCq - BAq = CAq = DCq$ ($- DAq$, per 7 e 2) $- BDq + 2BD \times BA - BAq$. Quare $BCq + BDq = DCq + 2BD \times BA$. In verbis, ſic, In triangulis obliquangulis, quadratum lateris ſubtendentis acutum angulum, minus eſt ſumma quadratorum laterum &c.



Probl: IV. Inventio 14 e 2: Nempe quadrati æqualis rectangulo $AB \times AD$. Eſto $AB + AD = 2BM$. Quare $AB + AD$ ſecatur æqualiter in M, & inæqualiter in A. Erit igitur per 5 e 2, $AB \times AD = BMq - AMq$. Jam ponatur $ACq = AB \times AD$: fiatque triangulum rectangulum MAC cujus hypotenuſa $CM = BM$ ſemiſummæ laterum; & baſis AM ſemidifferentiæ laterum: Cathetus erit AC latus quadrati quæſiti, per 48 e 1.



Inventio areæ trianguli plani.

Probl: V. Attulit ad me amicus quidam meus, vir doctus, Theorema de areâ trianguli plani; atque ut id examinarem, & demonſtratione munirem, poſtulavit.

Erat

Erat autem Theorema, prout memini (nam multi jam elapſi ſunt anni) hâc ferè formâ, licet non in iiſdem literis.

In triangulo plano cujus latera ſunt A,E,B; $\left\{\begin{array}{l}\frac{1}{8}BqEq-\frac{1}{16}Eqq\\ \frac{1}{8}EqAq-\frac{1}{16}Aqq\\ \frac{1}{8}AqBq-\frac{1}{16}Bqq\end{array}\right\}$ æquantur quadrato areæ trianguli.

Poſtquam aliquamdiu mecum cogitaſſem, occurrit mihi 17,c 18, Theor: 20, quod commodiſſimum huic nodo ſolvendo duxi. Nam ſi trianguli duo crura ſint A, E; & baſis B: inde liquebit, quod $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B$ in $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B$, in $\frac{1}{2}B+\frac{1}{2}A-\frac{1}{2}E$, in $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E$, æquatur quadrato areæ trianguli. Factâ igitur harum quatuor magnitudinum continuâ multiplicatione; prodibit $\frac{1}{8}AqEq+\frac{1}{8}AqBq+\frac{1}{8}EqBq-\frac{1}{16}Aqq-\frac{1}{16}Eqq-\frac{1}{16}Bqq$: Quod eſt ipſum Theorema propoſitum.

Atque hinc non ſolum poſtulato ſatisfeci; ſed etiam quatuor alia Theoremata effectu faciliora exhibui. Nam quia $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B=\frac{1}{2}Z+\frac{1}{2}B$: Et $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B=\frac{1}{2}Z-\frac{1}{2}B$. Et quia $\frac{1}{2}B+\frac{1}{2}A-\frac{1}{2}E=\frac{1}{2}B+\frac{1}{2}X$: Et $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E=\frac{1}{2}B-\frac{1}{2}X$: Erit $\frac{1}{2}Z+\frac{1}{2}B$ in $\frac{1}{2}Z-\frac{1}{2}B=\frac{1}{4}Zq-\frac{1}{4}Bq$. Et $\frac{1}{2}B+\frac{1}{2}X$ in $\frac{1}{2}B-\frac{1}{2}X=\frac{1}{4}Bq-\frac{1}{4}Xq$. Liquet igitur primò, $\frac{1}{4}Zq-\frac{1}{4}Bq$ in $\frac{1}{4}Bq-\frac{1}{4}Xq=Q$: areæ trianguli. In verbis ſic, Si quadrans differentiæ quadratorum ſummæ crurum & baſis ducatur in quadrantem differentiæ quadratorum baſis & differentiæ crurum; producta magnitudo æqualis erit quadrato Areæ trianguli.

Deinde

Deinde quia $\frac{1}{4}Zq-\frac{1}{4}Bq$ in $\frac{1}{4}Bq-\frac{1}{4}Xq = \frac{1}{16}ZqBq+\frac{1}{16}BqXq-\frac{1}{16}Bqq-\frac{1}{16}ZqXq$: Liquet ſecundò, $Zq+Xq-Bq$ in $\frac{1}{16}Bq$ mi $\frac{1}{16}ZqXq = Q$: Areæ trianguli.

Item quia $Zq+Xq = 2Z$, per 11, c 18: Et $ZqXq = Xq$, per 14, c 18: Liquet tertiò $2Z-Bq$ in $\frac{1}{16}Bq$ mi $Q: \frac{1}{4}X = Q$: Areæ trianguli.

Deniq; ex his comparatis, erit quartò $\left\{ \frac{2ZBq-Bqq-Xq}{16} \right. = Q$: Areæ tri.

Hæc poſteriora Theoremata verbis facilè enunciantur.

Probl: VI. Problematum circa Progreſſionem Arithmeticam ſolutio in viginti Propoſitionibus. Symbola verborum hæc ſint: α primus terminus minimus. ω ultimus maximus. T numerus terminorum. X differentia communis. Z ſumma omnium terminorum. Eſt igitur T--1 numerus differentiarum: ideoque $TX-X = \omega-\alpha$, ſumma differentiarum.

Datis tribus ex quinque illis α, ω, T, X, Z, invenire duo reliqua per viginti propoſitiones ſequentes (tot enim ſunt varietates) hoc ordine.

Datis	Quæruntur	Per Propoſiti:	
α, ω, T	Z & X	1 & 2	
α, ω, X	T & Z	3 & 4	
α, ω, Z	T & X	5 & 6	
α, T, X	ω & Z	7 & 8	
α, T, Z	ω & X	9 & 10	

Datis

Datis	Quæruntur	Per Propoſitio:
α, X, Z	ω & T	11 & 12
ω, T, X	α & Z	13 & 14
ω, T, Z	α & X	15 & 16
ω, X, Z	α & T	17 & 18
T, X, Z	α & ω	19 & 20

Prop: I. $T\omega + T\alpha = 2Z$.

II. $\frac{\omega - \alpha}{T - 1} = X$.

III. $\frac{\omega - \alpha}{X} + 1 = T$. per 2.

IIII. $\frac{\omega q - \alpha q}{X} + \omega + \alpha = 2Z$. per 1. 3.

V. $\frac{2Z}{\omega + \alpha} = T$. per 1.

VI. $\frac{\omega q - \alpha q}{2Z - \omega - \alpha} = X$. per 4.

VII. $TX - X + \alpha = \omega$. per 2.

VIII. $TX - X + 2\alpha$ in $X = 2Z$. per 1 & 7.

IX. $\frac{2Z - T\alpha}{T} = \omega$. per 1.

X. $\frac{2Z - 2\alpha}{Tq - T} = X$. per 8.

XI. $\sqrt{u}:\frac{1}{4}Xq+2ZX+\alpha q -X:$ minus $\frac{1}{2}X \Big\} = \omega$. per 4.

XII. Si $B = 2\alpha - X$, erit

$\sqrt{u}:\frac{Bq}{4Xq}-\frac{2Z}{X}:$ mi $\frac{B}{2X} = T$. per 8.

XIII. $\alpha + X - TX = \alpha$. per 7.

XIV. $2\omega + X - TX$ in $T = 2Z$. per 1 & 13.

XV. $\frac{2Z}{T} - \omega = \alpha$. per 9.

XVI. $\frac{2T\omega - 2Z}{Tq-T} = X$. per 14.

XVII. $\sqrt{u}:\frac{1}{4}Xq+\omega q+X\omega -2ZX:$ plus $\frac{1}{2}X \Big\} = \alpha$. per 4.

XVIII. $\frac{2\omega+X}{2X}$ minus $\sqrt{u}:\frac{4\omega q+4X\omega+Xq-2Z4X}{4Xq} \Big\} = T$. per 14.

XIX. $\frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \alpha$. per 10.

XX. $\frac{2Z}{T} + \frac{TX}{2} - \frac{X}{2} = \omega$. per 16.

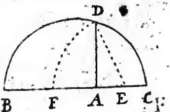
Probl: VII.

Probl: VII. Euclides. 11 e 2, docuit ſecare lineam datam, ſic, ut rectangulum ſub tota & minore ſegmento, æquetur quadrato majoris ſegmenti: quæ ſectio eſt penè divina. Proponatur jam illud problema generaliter; ut data linea AB ita ſecetur, ut rectangulum ſub tota AB, & minore ſegmento, ad quadratum majoris ſegmenti, rationem quamcunque poſſibilem datam habeat: puta R ad S.

Primo fiat R. S :: AB. AC: qui quartus ſit proportionalis. tum pro majore ſegmento ponatur A: minus ſegmentum erit AB−A: quod ductum in AB, dabit rectangulum ABq−AB×A. Erit igitur AB.AC :: ABq−AB×A. Aq. Ideoque per 3 cap:6, ABq×AC−AB×AC×A = AB×Aq. Et diviſis omnibus per AB, erit AB×AC−AC×A = Aq: vel Aq+AC×A = AB×AC. Et per 9 cap:16, invenitur $\sqrt{}$u: $\frac{1}{4}$ ACq+AB×AC:−$\frac{1}{2}$AC = A.

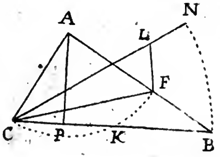
Hoc theorema inventum, verbis ſic enuntiatur: Si ad quadratum ſemiſſis quarti proportionalis, adjungatur rectangulum ſub linea recta data, & quarto illo proportionali; Et ex latere quadrato ſummæ tollatur ſemis quarti proportionalis: Reliquum erit ſegmentum majus.

Geometricè ſic. Statuantur AB & AC in directum: Et diametro BC fiat ſemicirculus: Et ſuper BC in puncto A, erigatur perpendicularis AD, ſecans ſemicirculum in D. tum biſecta AC in E, menſuretur



EF $=$ ED. Dico lineam AB ſic ſecari in puncto F ut ſit R.S :: AB×BF. AFq. Nam AC×AF+AC×BF $=$ AC×AB $=$ ADq $=$ CF×AF, per 6 e 2, $=$ AC×AF+AFq. Quare AC×BF $=$ AFq. Atqui AB.AC :: AB×BF.AC×BF. Ergo,

Prob: VIII. Dato latere alterutro trianguli rectanguli (in quo perpendicularis ex angulo recto ſecat hypotenuſam) una cum BK differentia ſegmentorum hypotenuſæ, invenire tum hypotenuſam, tum triangulum ipſum. Primò detur latus minus CA. Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC: in quo è vertice in hypotenuſam demittatur perpendicularis AP, ſecans hypotenuſam in BP & CP ſegmenta. Eſt autem CP $=\frac{BC-BK}{2}$ Quia eſt BC. CA :: CA. $\frac{BC-BK}{2}$ erit $\frac{BCq-BC\times BK}{2}=$ CAq: vel BCq —BK×BC $=$ 2CAq. quare per 9 c 16, $\sqrt{q:\frac{1}{4}BKq+2CAq:}+\frac{1}{2}BK=BC$. Enunciatur autem hoc theorema verbis ſic: Si quadratum ſemidifferentiæ ſegmentorum hypotenuſæ addatur duobus quadratis lateris dati; & aggregati latus quadratum augeatur



ipſa ſemi-differentia: tota aucta æqualis erit hypotenuſæ.

Geo-

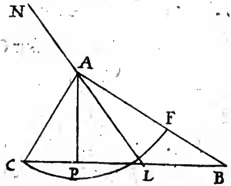
Geometricè ſic. Ducatur CF: ipſique perpendicularis $FL = \frac{BK.}{2}$ & extendatur CL ad N, ut $LN = \frac{1}{2}BK$. Erit $CN = BC$. quare inſcribatur circulo CK $= CN - BK$: & producatur, &c. Nam $CFq = 2CAq$. & $CLq = 2CAq + \frac{1}{4}BKq$. Ergo,

Si verò detur majus latus BA: hujusmodi invenietur æquatio, $\sqrt{q}: \frac{1}{4}BKq + 2BAq: - \frac{1}{2}BK = BC$. ſumpta $\frac{BC+BK}{2}$ pro PB.

Et modus geometricus priori non abſimilis.

Prob. IX. Datis differentia laterum trianguli rectanguli BF, & perpendiculari AP ab angulo recto in hypotenuſam: invenire tum hypotenuſam, tum triangulum ipſum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam per 7 e 2. $2BA \times AF + BFq = BAq + AFq$, Ideoque $BFq = (ABq + AFq$, hoc eſt) $BCq - (BA \times 2CA$. hoc eſt) $BC \times 2AP$, quia BC. CA :: BA. AP. Erit $BCq - 2AP \times BC = BFq$. quare per 9 c 16. $\sqrt{q}: APq + BFq: + AP = BC$.



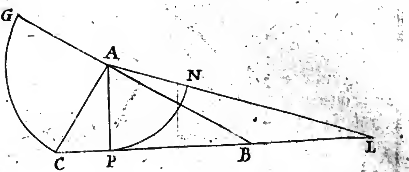
Enunciatur autem hoc theorema verbis ſic: Si quadrato perpendicularis addatur quadratum differentiæ laterum; & aggregati latus quadratum augeatur ipſa perpendiculari: tota aucta æqulis erit hypotenuſæ.

Geometricè ſic. Fiat PL=BF. Et extendatur LA ad N, ut AN=AP. Erit LN=BC. Diametro igitur BC deſcribatur ſemicirculus; in quo ſtatuatur perpendicularis æqualis datæ AP. Et ducantur BA, & CA.

Probl: X. Datis ſumma laterum trianguli rectanguli, BG, & perpendiculari ab angulo recto in hypotenuſam, AP: invenire tum hypotenuſam, tum triangulum ipſum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam per 4 e 2, BGq =(BAq+GAq, hoc eſt) BCq+ (2BA×CA, hoc eſt) 2AP×BC, quia BC. CA :: BA. AP. Erit BCq +2AP×BC =BGq. quare per 9 c 16, √q: APq +BGq: −AP=BC.

Enunciatur



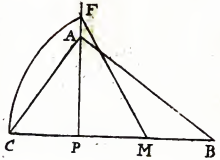
Enunciatur autem hoc theorema ſic. Si quadrato perpendicularis addatur quadratum ſummæ laterum; & aggregati latus quadratum minuatur ipſa perpendiculari: linea reliqua æqualis erit hypotenuſæ.

Geometricè ſic. Fiat PL = BG & ducatur AL: ex qua abſcindatur AN = AP. Erit LN = BC. Diametro igitur BC deſcribatur ſemicirculus, &c.

Probl: XI. Datis trianguli rectanguli latere alterutro, CA, & alterno ſegmento hypotenuſæ BP: invenire tum alterum ſegmentum, tum ipſum triangulum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam eſt BP + CP. CA :: CA. CP. Erit BP×CP + CPq = CAq. quare per *9 c 16*, $\sqrt{q}: \frac{1}{4}$BPq + CAq: $- \frac{1}{2}$ BP = CP.

Enuntiatur autem hoc theorema verbis ſic. Si quadrato ſemiſſis ſegmenti hypotenuſæ addatur quadratum lateris dati; & aggregati latus quadratum minuatur



ipſo ſemiſſe: linea reliqua erit alterum hypotenuſæ ſegmentum.

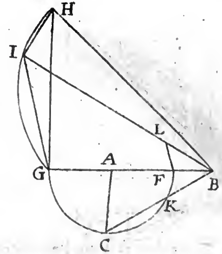
Geometricè ſic. Statuantur ad angulos rectos BP & PF = CA & biſecta BP in M, ducatur MF: cui menſuretur æqualis MC. Inventum eſt igitur CP alterum ſegmentum: & BC tota hypotenuſa. Diametro BC deſcribatur ſemicirculus: in quo inſcribantur CA, & BA.

Probl: XII. Datis trianguli rectanguli differentia ſegmentorum hypotenuſæ BK, & ſumma laterum, BG: invenire tum differentiam laterum, tum hypotenuſam, tum ipſum triangulum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam eſt BG.BK :: BC. BF: eſt etiam BGq. BKq :: (BCq, hoc eſt) BAq + CAq. BFq. Item 2BGq-BKq. BKq :: (2BAq + 2CAq — BFq hoc eſt) BGq. BFq: Nam per 8 c 18 2BAq + 2CAq = BGq + BFq. quare √q: 2BGq — BKq. BG :: BK. BF :: BG BC.

Enunciatur autem hoc theorema verbis ſic. Si

Si è quadrato ſummæ laterum duplicato tollatur quadratū differentiæ ſegmentorum hypotenuſæ: Erit ut latus quadratum reliqui, ad ſummam laterum; ſic differentia ſegmentorū hypotenuſæ, ad differentiam laterum, & ſic ſumma laterum ad hypotenuſam.

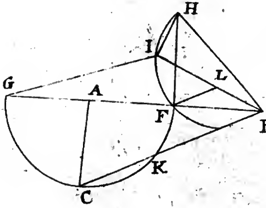


Geometricè ſic. Statuantur ad angulos rectos BG & GH=BG. tum diametro BH deſcribatur ſemicirculus: in quo inſcribatur HI=BK: & ducatur BI. Eſt igitur BI=√q: 2BGq—BKq. fiat etiam BL =BK. Ducatur GI: eique parallela LF. Ergo inventa eſt BF differentia laterum.

Probl: XIII. Datis trianguli rectanguli differentia ſegmentorum hypotenuſæ BK, & differentia laterum BF: invenire tum ſummam laterum, tum hypotenuſam, tum ipſum triangulum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam eſt BF. BK :: BC. BG: eſt etiam BFq. BKq :: (BCq, hoc eſt) BAq +CAq. BGq. Item 2BFq—BKq. BKq :: (2BAq +2CAq—BGq, hoc eſt) BFq, BGq: Nam per 8 c 18 2BAq+2CAq=BGq+BFq: √q: 2BFq—BKq. BF:: BK. BG:: BF. BC.

Enuncia-

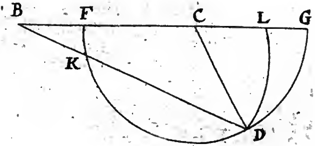


Enunciatur autem hoc theorema verbis ſic. Si è quadrato differentiæ laterum duplicato tollatur quadratum differentiæ ſegmentorum hypotenuſæ: Erit ut latus quadratum reliqui, ad differentiam laterum; ſic differentia ſegmentorum hypotenuſæ, ad ſummam laterum: & ſic differentia laterum, ad hypotenuſam.

Geometricè ſic. Statuantur ad angulos rectos BF & FH=BF. tum diametro BH deſcribatur ſemicirculus, in quo inſcribatur BI=BK: & ducatur HI. Eſt igitur HI=$\surd$q:2BFq—BKq: fiat BL=HI. Ducatur FL: eique parallela IG. Ergo inventa eſt BG ſumma laterum.

Probl. XIV. Datis trianguli plani cujuſcunque differentia laterum FB, differentia ſegmentorum baſis BK, & differentia inter majus latus & baſem CL: invenire tum baſem, tum ſummam laterum, tum ipſum triangulum. Et primò ſit exceſſus penes baſem. Puta factum

factum eſſe quod poſtulatur: ſitque triangulum BCD. Quoniam eſt FB.BK :: BD. $\frac{BK \times BD}{BF} = BG$, per 17 c 18, Th: 16. Erit $\frac{BK \times BD - BFq}{2BF} = CF$. adde BF,& $\frac{BK \times BD + BFq}{2BF} = BC$. tolle hanc ex BD, & $\frac{2BF \times BD - BK \times BD - BFq}{2BF} = \frac{2BF \times CL:}{2BF}$ Quare $2BF \times BD - BK \times BD = 2BF \times CL + BFq$. & per 3 c 6. $2BF - BK.2CL + BF :: BF.BD :: BK.BG$.



Enunciatur autem hoc theorema verbis ſic. ut differentia inter differentiam laterum duplicatam,& differentiam ſegmentorum baſis, eſt ad aggregatum differentiæ inter majus latus & baſim duplicatæ, & differentiæ laterum; ſic differentia laterum, ad baſem: & ſic differentia ſegmentorum baſis, ad ſummam laterum.

Geometrica praxis facilior eſt, quàm ut neceſſe ſit apponi.

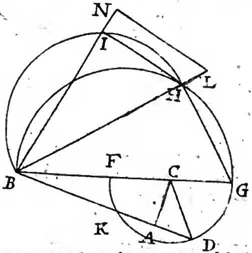
Si verò exceſſus fuerit penes majus latus : theorema erit, BK - 2BF. 2CL-BF :: BF. BD::BK.BG.

Hujus theorematis inveſtigationem; & problematis quo è datis trianguli plani cujuſcunque ſumma laterum BG, differentia ſegmentorum baſis BK, & differentia inter majus latus & baſem CL. poſtulatur invenire tum baſem, tum differentiam laterum, ſolutionem, omitto : ut habeant ſtudioſi analyſeos, quo ſoliertiam ſuam exerceant.

Probl:XV. Datis trianguli plani cujuſcunque ſumma laterum BG, differentia ſegmentorum baſis BK,& perpendiculari CA : invenire tum baſem, tum differentiam laterum,tum ipſum triangulum. Puta factum eſſe quod poſtulatur: ſitque triangulum BCD. Quoniam per 17 c 18 Th: 16. BG. BD :: BK. BF. Et per 5 c 18, DKq=BDq + BKq-2BK×BD. Et per 47 e 1 (4 ADq hoc eſt) DKq + 4CAq = (4CDq, hoc eſt) FGq. Erit BDq+BKq -2BK×BD+4CAq =FGq. Tolle FG ex BG:& BG-√q: BDq+BKq -2BK×BD+4CAq:=BF. Quare erit,

BG. BD :: BK. BG—√q: BDq+BKq—2BK×BD +4CAq. Et per 3 c 6, BK×BD=BGq—√q:BGq ×BDq+BGq×BKq—BGq×2BK×BD+BGq×4CAq. Eſt igitur per 8 c 16 Q: BGq—BK×BD, hoc eſt, BGqq—BGq×2BK×BD + BKq×BDq = BGq ×BDq+BGq×BKq-BGq×2BK×BD+BGq×4CAq. Ideóque BGq×BDq-BKq×BDq=BGqq-BGq ×BKq—BGq×4CAq. vel etiam, BGq—BKq in

in BDq=BGq—BKq-4CAq in BGq. Ergo √q: BGq-BKq.√q: BGq-BKq — 4CAq:: BG.BD:: BK. BF.



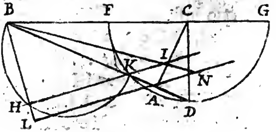
Enunciatur autem hoc theorema verbis ſic, ut latus quadratum differentiæ inter quadrata ſummæ laterum, & differentiæ ſegmentorum baſis, eſt ad latus quadratum ejuſdem differentiæ multatæ quadrato perpendiculi duplicati; ſic ſumma laterum, ad baſem: & ſic differentia ſegmentorum baſis, ad differentiam laterum.

Geometricè ſic. Diametro BG deſcribatur ſemicirculus: in quo inſcribatur GH=BK: & BH. Eſt igitur BH=√q: BGq—BKq. Rurſus diametro BH deſcribatur ſemicirculus: in quo inſcribatur HI=2CA:& BI. Eſt igitur BI=√q:BGq-BKq —4CAq. Fiat BL=BG:& ab L ducatur LN parallela ipſi HI, concurrens cum BI producta in N. Ergo inventa eſt BN=BD.

Probl:

Probl: XVI. Datis trianguli plani cujuſcunque differentia laterum BF, differentia ſegmentorum baſis BK, & perpendiculari CA : invenire tum baſem, tum ſummam laterum, tum ipſum triangulum.

Puta factum eſſe quod poſtulatur : ſitque triangulum BCD. Quoniam eſt BG.BD::BK.BF. Et $DKq = BDq + BKq - 2BK \times BD$, per 5 c 18. Et per 47 e 1, ($4ADq$, hoc eſt) $DKq + 4CAq =$ ($4CDq$, hoc eſt) FGq. Erit $BDq + BKq - 2BK \times BD + 4CAq = FGq$. Adde FG ad BF : Et $BF + \sqrt{q} : BDq + BKq - 2BK \times BD + 4CAq = BG$. Quare BF. BD :: BK. $BF + \sqrt{q} : BDq + BKq - 2BK \times BD + 4CAq$. Item $BK \times BD = BFq + \sqrt{q} : BFq \times BDq + BFq \times BKq - BFq \times 2BK \times BD + BFq \times 4CAq$. Eſt igitur Q: $BK \times BD - BFq$, hoc eſt, $BKq \times BDq - BFq \times 2BK \times BD + BFqq = BFq \times BDq + BFq \times BKq - BFq \times 2BK \times BD + BFq \times 4CAq$. Ideoque $BKq \times BDq - BFq \times BDq = BFq \times BKq - BFqq + BFq \times 4CAq$. vel etiam $BKq - BFq$ in $BDq = BKq - BFq + 4CAq$ in BFq. Ergo $\sqrt{q} : BKq - BFq . \sqrt{q} : BKq - BFq + 4CAq$: : BF.BD::BK.BG.



Enunciatur

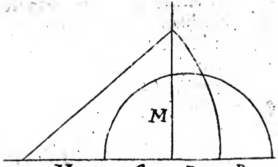
Enunciatur autem hoc theorema verbis ſic. ut latus quadratum differentiæ inter quadrata differentiæ ſegmentorum baſis, & differentiæ laterum, eſt ad latus quadratum ejuſdem differentiæ auctæ quadrato perpendiculi duplicati; ſic differentia laterum, ad baſem: & ſic differentia ſegmentorum baſis, ad ſummam laterum.

Geometricè ſic. Diametro BK deſcribatur circulus: in quo inſcribatur KH=BF: & BH. Eſt igitur $BH=\sqrt{q}: BKq - BFq$. Fiat BHL=BF: & HKI =2CA. Ducatur BI. Eſt igitur $BI=\sqrt{q}: BKq - BFq+4CAq$. Ducatur etiam LN parallela ipſi HI, concurrens cum BI producta in N. Ergo inventa eſt BN=BD.

Probl: XVII. Datis in triangulo rectangulo differentia inter baſem & hypotenuſam B, & differentia inter cathetum & hypotenuſam C: invenire tum hypotenuſam, tum ipſum triangulum.

Pro hypotenuſa ponatur A. Baſis erit A—B. & Cathetus A—C. & per 47 e 1, Cathetus eſt $\sqrt{q}: 2BA - Bq$. Quare $\sqrt{q}: 2BA - Bq = A - C$. Et $2BA - Bq = Aq - 2CA + Cq$. vel 2B+2C in A mi $Aq = Bq + Cq$. Ergo per 9 c 16, $B+C+\sqrt{q}: 2BC = A$, hypotenuſæ.

Enunciatur



Enunciatur autem hoc theorema verbis ſic. Aggregatum utriuſque differentiæ (tum baſis tum catheti) ab hypotenuſa. unâ cum √q duplicis rectanguli ſub ipſis differentiis, æquatur hypotenuſæ.

Geometricè ſic. Ducatur linea infinita in qua menſurentur B, B, & C. hac diametro fiat ſemicirculus. Et in communi B & C termino ſtatuatur ad angulos rectos linea M. Eſt igitur Mq = 2BC. menſuretur etiam M in linea infinita poſt C. Et ſemidiametro M+C+B deſcribatur arcus donec concurrat cum linea M perpendiculari producta. tum à puncto concurſus ad centrum illius arcus ducatur linea pro hypotenuſa. Et deſcriptum erit triangulum rectangulum quæſitum.

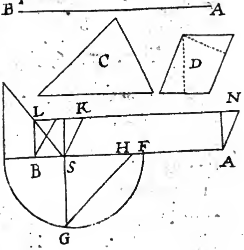
Probl: XVIII. Ad datam rectam lineam AB, dato rectilineo C æquale parallelogrammum adplicare, deficiens figura parallelogramma, quæ ſimilis alteri parallelogrammo D dato. Oportet autem datum rectilineum non majus eſſe eo, quod ad dimidium adplicatur. Prop: eſt 28 è 6.

In

In parallelogrammo D, notetur lineis perpendicularibus ejus Altitudo R, & Latitudo S: nec refert utra ex ipſis ſtatuatur major.

Ponatur latus parallelogrammi quæſiti A : portio ablatitia erit AB-A. Fiat S.R :: AB-A. $\frac{AB*R-R*A}{S}$ altitudo parallelogrammi quæſiti: Ducatur in A latus: eritque $\frac{AB*R*A-R*Aq}{S}$ = C: vel AB*A-Aq = $\frac{C*S}{R}$ Et per 9, Cap: 16, $\frac{AB}{2}+\sqrt{q}:\frac{ABq}{4}-\frac{C*S}{R}:=A.$

Quod Theorema verbis enuntiatur ſic. Si rectilineum C datum ducatur in latitudinem parallelogrammi D; & factus dividatur per altitudinem : & quotus ex quadrato ſemiſſis lineæ AB datæ auferatur : latus quadratum reliqui auctum eodem ſemiſſe, erit latus parallelogrammi quæſiti.



Geometricè ſic: Fiat ER=$\sqrt{q}$C. Tum R.S :: ER. ES=$\frac{\sqrt{C}}{R}$ Statuantur ER & ES ad angulos rectos : Sumptaque SF=ER, diametro EF deſcribatur ſemicirculus: in quo erectâ perpendiculari SG erit SGq=$\frac{C*S}{R}$.

Ex G puncto menſuretur $GH = \frac{1}{2} AB = HB$: erit $HS = \sqrt{:} \frac{ABq}{4} - \frac{C \times S}{R}$: cui ſi adjungas $HA = \frac{1}{2} AB$; erit AS latus parallelogrammi quæſiti. Et $BS = AB - A$ portioni ablatitiæ. Et BL parallela lineæ ER, erit altitudo. Ergò parallelogrammum quæſitum eſt ASKN, factum ipſi D æquiangulum.

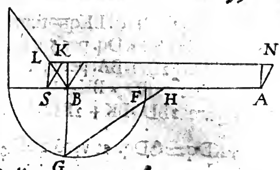
Probl: XIX. Ad datam rectam lineam AB, dato rectilineo C æquale parallelogrammum adplicare, excedens figura parallelogramma, quæ ſimilis ſit alteri parallelogrammo D dato, Prop: eſt *29 e 6*.

In parallelogrammo D notetur Altitudo & Latitudo, ſicut in præcedente.

Ponatur latus parallelogrammi quæſiti A : Portio adjectitia erit $A - AB$. Fiat $S . R :: A - AB . \frac{R \times A - AB \times R}{S}$ altitudo parallelogrammi quæſiti. Ducatur in A latus: Eritque $\frac{R \times Aq - AB \times R \times A}{S} = C$. vel $Aq - AB \times A = \frac{C \times S}{R}$ Et per *9 c 16*, $\sqrt{q:} \frac{ABq}{4} + \frac{C \times S}{R} : + \frac{AB}{2} = A$.

Quod Theorema verbis enunciatur ſic. Si rectilineum datum C ducatur in latitudinem parallelogrammi D; & factus per altitudinem dividatur: & quotus addatur quadrato ſemiſſis lineæ AB datæ: Latus quadratum aggregati, auctum eodem ſemiſſe, erit latus parallelogrammi quæſiti.

Geometricè



Geometricè ſic. Fiat $ER = \sqrt{q}C$. Tum R.S :: ER.EB $= \frac{\sqrt{C \times S}}{R}$. Statuantur ER & EB ad angulos rectos: Sumptaque BF = ER, diametro EF deſcribatur ſemicirculus: in quo erecta perpendiculari BG, erit $BGq = \frac{C \times S}{R}$. Eſto $BH = \frac{1}{2} AB = AH$. Et ducatur $GH = \sqrt{q}: \frac{ABq}{4} + \frac{C \times S}{R} = HS$: Eſt igitur AS = A lateri parallelogrammi quæſiti: Et BS = A - AB portioni adjectitiæ. Et altitudo erit SL parallela lineæ ER. Ergo parallelogrammum quæſitum eſt, ASKN factum ipſi D æquiangulum.

Probl: XX. Datis trianguli plani cujuſcunque duobus lateribus BC BD, cum angulo B intercepto: invenire tertium latus, vel datis tribus lateribus: invenire angulum B, uni ipſorum oppoſitum.

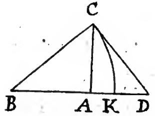
Eſto factum quod poſtulatur: ſitque triangulum BCD. Centro B, ſemidiametro BC, deſcribatur arcus CK; & perpendicularis CA. Eſt igitur KD differentia laterum: & AK ſimilis ſinui verſo anguli B. Nam Rad. $s v$B :: BK. AK. Eſtque $AK = \frac{BK \times s v B}{Rad.}$. Eſt autem etiam $AK = BK \mp BA$: ut ex ſchematibus comparatis liquet.

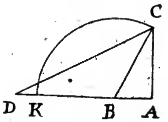
Et quia $BDq + BKq =$ tum

$$\left\{ \begin{array}{l} 2BD \times BK + KDq. \text{ per 5 c 18} \\ CDq \pm 2BD \times BA. \text{ per 2,3, c 19} \end{array} \right\}$$

Erit $2BD \times BK + KDq = CDq \pm 2BD \times BA$. Quare $2BD \times BK \mp 2BD \times BA$. hoc eſt, $2BD \times AK$ $+ KDq = CDq$. at verò $2BD \times AK = \frac{2BD \times BC \times svB}{Rad:}$

Ergo $\frac{2BD \times BC \times svB}{Rad:} + KDq = CDq$. Quod eſt theorema primum. Et $\frac{CDq - KDq \text{ in } Rad:}{2BD \times BC} = sv\, B$. Quod eſt theorema ſecundum.





Enunciatur quidem verbis primum theorema ſic: Si duplicatum rectangulum ſub lateribus datis ducatur in ſinum verſum anguli intercepti: & factus dividatur per Radium: Quotus auctus quadrato differentiæ laterum æqualis erit quadrato tertii lateris.

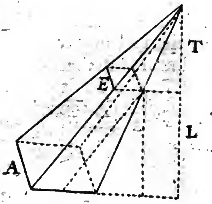
Secundum verò ſic: Si differentia quadratorum lateris oppoſiti,& differentiæ laterum,ducatur in Radium; & factus dividatur per duplicatum rectangulum ſub lateri-

lateribus continentibus: Quotus æqualis erit ſinui verſo anguli quæſiti.

Probl: XXI. Datis fruſti Pyramidis utraque baſe Aq, Eq, & altitudine L: invenire menſuram fruſti.

Prænoſcendum eſt ex 7 & 10 e 12. quod parallelepipedon æquatur tribus pyramidibus; Et Cylindrus æquatur tribus conis, ejuſdem baſis & altitudinis.

Eſtque altitudo pyramidis abſciſſæ (T) primò quærenda, ſic, A—E. E :: L. T. Quare $\frac{LE}{X} = T$. Et altitudo totius pyramidis eſt L+T. Item pyramis toto tripla eſt AqL+AqT. Et pyramis abſciſſa tripla eſt Eq T. Ergo triplum fruſtrum pyramidis eſt AqL +AqT−EqT.



Hoc theorema oſtendit unum modum commenſurandi fruſtum pyramidis: Enunciatur autem verbis ſic.

Si ſolidum ſub baſe majore & tota altitudine multetur ſolido ſub baſe minore & altitudine pyramidis abſciſſæ: reliqui triens æqualis erit fruſto.

Rurſus quia 2 c 11. $Aq-Eq = ZX$: & $T = \frac{LE}{X}$ Erit $AqL+$ (ZEL, hoc eſt per 3 è 2) $AEL. + EqL = AqL + AqT - EqT$. Ergo triplex fruſtum pyramidis eſt etiam $Aq+Eq+AE$ in L. hoc theorema docet alterum modum commenſurandi fruſti : enunciatur autem verbis ſic.

Si aggregatum utriuſque baſis fruſti pyramidis, & mediæ inter ipſas proportionalis, ducatur in altitudinem fruſti: facti triens æqualis erit fruſto.

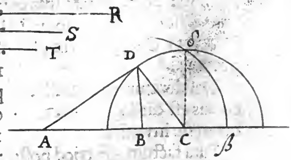
Item quia per 2 c 11, $2Aq+2Eq = Zq+Xq$: Erit $ZqL+XqL+2AEL$ æquale ſex fruſtis. at per 11 e 18. $Xq+2AE = Z$. Ergo $Zq+Z$ in L æquale eſt ſex fruſtis pyramidis. Atque hoc Theorema docet tertium modum commenſurandi fruſti pyramidis. Enuntiatur autem verbis ſic. Si ad aggregatum baſium addatur quadratum aggregati laterum quadratorum utriuſque baſis, & ſumma eorundem ducatur in altitudinem fruſti: facti ſextans æqualis erit fruſto.

At verò ſi quæſtio ſit de commenſurando fruſto Coni. Quia juxta Archimedæum inventum, ſemiperipheria circuli æqualis eſt $\frac{22}{7}$ Radii ferè : vel magis accuratè $\frac{355}{113}$ Rad :: Erit area circuli $\frac{355}{113}$ Rad: q. Et 113 . 355 :: Rad:q. area circuli. Quare Theorema primum de commenſurando fruſto coni, eſt $\frac{355}{452}$ AqL $+\frac{355}{452}$ AqT $-\frac{355}{452}$ EqT, æquatur triplo fruſto. Secundum eſt $\frac{355}{452}$ Aq$+\frac{355}{452}$Eq$+\frac{355}{452}$ AE in L, æquatur triplo fruſto. Tertium eſt $\frac{355}{452}$ Zq$+\frac{355}{452}$ Z in L, æquatur ſextuplo fruſto Coni.

Probl:

Probl: XXII. Problema Apolonii Pergæi ἐν ἀναλυομένῳ τόπῳ Datis in plano duobus punctis A, B, deſcribere circulum, in cujus circumferentiam rectæ lineæ, AD, BD, ab iiſdem punctis ductæ, datam habeant rationem R ad S.

Puta factum eſſe quod quæritur: ſitque circuli quæſiti centrum C in eadem recta linea cum punctis A, B; & ſemidiameter CD. Fiat R.S :: S.T. Quia triangula duo ACD, DCB (ubicunque ſumitur punctum D) ſunt ut AC ad BC: Et latera DA, DB, communi angulo C ſimiliter oppoſita, ſunt in ratione R ad S: & latus CD utrique commune: Non difficile erit concipere triangula ipſa ACD, DCB eſſe ſimilia. Quare R, S :: DA. DB :: AC. DC :: DC. BC. Et per 1 c 15, AC. BC :: Rq. Sq :: R.T. Si igitur pro BC ponatur A: Erit AB+A. A :: R.T. Et AB×T+T×A = R×A: vel $\frac{AB \times T}{R - T} = A$. Denique $\sqrt{}$: AC×BC: = DC.



Quæ enuntiatur verbis ſic. Si punctorum intervallum ducatur in tertium rationis datæ terminis proportionalem: & factus dividatur per exceſſum termini primi ſupra tertium: Quotus æqualis erit diſtantiæ puncti citerioris à centro. Et latus quadra-

 tum

tum rectanguli ſub utraque diſtantia à centro, æquatur ſemidiametro. Geometrica effectio facillima eſt.

Probl: XXIII. Datis dolii, ſive vaſis vinarii, longitudine interna 2CL, & ſemidiametris tum medii CB, tum baſis LD: invenire dolii ipſius capacitatem. Eſt quidem dolium fruſtum ſphæroideos, quæ fit revolutione ſemiſſis ellipſeos ſuper diametrum ſuam tranſverſam ſive axim. Ad menſuram autem fruſti inveniendam, tum totius ſphæroideos, tum abſciſſarum portionum menſuras ſciri oportet: harum enim menſurarum differentia eſt menſura fruſti.

Soliditas totius ſphæroideos eſt $\frac{355}{113}$ BCq in $\frac{2}{3}$ IK: qui duplus eſt conus baſis BCB, & altitudinis IK: Archim: de conoid: & ſpæroid: prop. 29.

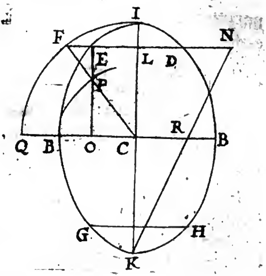
Soliditas verò portionis IED abſciſſæ habetur ſic: LK.LK+KC::$\frac{355}{113}$ LDq in $\frac{1}{3}$ LI. Soliditas quæſita. Ibid. prop. 31.

Deſideratur autem adhuc (qui hujus negotii præcipuus eſt cardo) diameter tranſverſa ſive axis IK: quem ſic invenies.

Puta factum eſſe quod poſtulatur: Et deſcribatur ellipſis: & reliqua; ſicut in ſchemate. Et fiat CK. CB::CB.$\frac{CBq}{CK}$=CR, quod eſt ſemilatus rectum per 13 l 1 conic: Apoll. Iterum fiat CK. $\frac{CBq}{CK}$ :: CK+CL. $\frac{CBq \text{ in } CK+CL}{CKq}$=LN. ducatur in IL, hoc eſt CK—CL (quod idem eſt ac ſi ducatur $\frac{CBq}{CKq}$ in

CKq−CLq per 11 c 18) fietque $\frac{CBq \times CKq - CBq \times CLq}{CKq}$ $=$ LEq. per 13 l 1 conic: Apoll. Ergo $\sqrt{q}\,\frac{CBq \times CLq}{CBq - LEq}$ $=$ CK, ſemi-axi; hoc eſt $\frac{CB \times CL}{OP} = CK$.

Quod theorema verbis enunciatur ſic. Si quadratum ſemidiametri medii dolii ducatur in quadratum dimidiatæ longitudinis: & factus dividatur per differentiam quadratorum à ſemidiametris medii & baſis: quoti latus quadratum erit ſemiaxis ſphæroideos.



Geometricè ſic. Ducatur EO parallela axi. Et ſemidiametro CP = CB fiat arcus ſecans ipſam EO in P. continuetur CP donec concurrat cum baſe LE producta in F. Erit CF æqualis ſemiaxi quæſito.

Conſectarium. Atque hinc patet meridianos in Analemmate eſſe veras Ellipſes, verbi gratia, cogitetur quadrans Analemmatis CIFQ: in quo deſcripta ſit Ellipſis IEB. Dico eundem eſſe Meridianum. Nam

cum

cum CQ ſit quadrans Æquinoctialis, & FL quadrans paralleli: ſitque meridianorum proprium ſecare Æquinoctialem, & omnes circulos ipſi parallelos, in ſegmenta ſimilia, per 10, l 2 Theod: de ſphæra. Si igitur conſtiterit eſſe CQ. CB :: LF. LE: Ellipſis IEB ſecans ipſos erit meridianus. At verò CF = CQ: & CP = CB: & OC = LE: Eſtque CF. CP :: LF. OC. Ergo,

Probl: XXIV. Datis trianguli rectanguli hypotenuſa BC, & CM mediâ proportionali inter baſem & cathetum; invenire triangulum.

Puta factum eſſe quod poſtulatur: ſitque triangulum rectangulum BAC. Quoniam baſis eſt BA, Cathetus erit $\sqrt{q}$: BCq - BAq: & rectangulum ſub ipſis $\sqrt{q}$: BCq × BAq - BAqq: cujus latus quadratum eſt $\sqrt{qq}$: BCq × BAq - BAqq: media proportionalis inter baſem & Cathetum.

Item quoniam Cathetus eſt CA, Baſis erit $\sqrt{q}$: BCq - CAq. Et rectangulum ſub ipſis, $\sqrt{q}$: BCq × CAq - CAqq: cujus latus quadratum eſt $\sqrt{qq}$: BCq × CAq - CAqq: media proportionalis inter baſem & Cathetū.

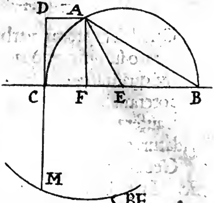
Quare BCq × BAq - BAqq = CMqq. Et
BCq × CAq - CAqq = CMqq.

$$\text{Ergo per } 9c16,\ \tfrac{1}{2}BCq \pm \sqrt{q}:\tfrac{1}{4}BCqq - CMqq. = \left\{\begin{matrix} BAq \\ CAq \end{matrix}\right.$$

Quod Theorema enuntiatur verbis ſic. Si dimidiato hypotenuſæ quadrato, latus quadratum exceſſus quadrantis quadrato-quadrati hypotenuſæ ſupra quadrato-quadratum medii proportionalis inter baſem & Cathetum, addatur; aggregatum erit baſis quadratum

dratum: ſin auferatur, reliquum erit quadratū Catheti.

Geometricè ſic. Diametro BC, & centro E medio, deſcribatur ſemicirculus: Tum fiat BC.CM::CM.CD =AF perpendic: intra ſemicirculum. Eſt igitur $BC \times AF = CMq$. compleatur triangulum BAC. Nam $\frac{1}{4}BCq(AEq) - AFq = EFq$.



$$\text{Quare } \tfrac{1}{2}BC \pm (EF)\ \sqrt{q}{:}\tfrac{1}{4}BCq - AFq{:} = \begin{cases} BF. \\ CF. \end{cases}$$

Ducantur omnia in BC: fietque

$$\tfrac{1}{2}BCq + \sqrt{q}{:}\tfrac{1}{4}BCqq - (BCq \times AFq)CMqq{:} = \begin{cases} BC \times BF = BAq. \\ BC \times CF = CAq. \end{cases}$$

Probl: XXV. Datis trianguli rectanguli baſe BA, & AM mediâ proportionali inter hypotenuſam & Cathetum, invenire triangulum.

Puta factum eſſe quod poſtulatur : ſitque triangulum rectangulum BAC. Quoniam Cathetus eſt CA, hypotenuſa erit $\sqrt{q}{:}\ BAq + CAq$: Et media inter ipſas proportionalis $\sqrt{qq}{:}\ CAqq + BAq \times CAq$.

Item quoniam hypotenuſa eſt BC, cathetus erit $\sqrt{q}\ BCq - BAq$: Et media inter ipſas proportionalis $\sqrt{qq}{:}\ BCqq - BAq \times BCq$.

Quare $CAqq + BAq \times CAq = AMqq$. Et

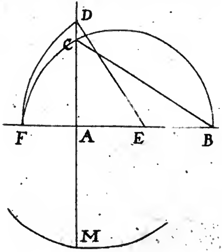
$BCqq - BAq \times CAq = AMqq$. Ergò per 9c16

$\sqrt{q}{:}$

$\sqrt{q}: \frac{1}{4}BAqq + AMqq: \mp \frac{1}{2}BAq = \begin{cases} CAq. \\ BCq. \end{cases}$

Quod Theorema verbis enuntiatur ſic. Si lateri quadrato ſummæ ex quadrante quadrato-quadrati baſis, & quadrato-quadrati mediæ proportionalis inter hypotenuſam & Cathetum, dimidiatum baſis quadratum auferatur, reliquum erit Catheti quadratum : ſin addatur, aggregatum erit quadratum hypotenuſæ.

Geometricè ſic. Fiat BA. AM :: AM.AD perpendic: eſt igitur BA*AD =MAq. ex medio baſis puncto E ad perpendicularem AD, ducatur ED=EF Et diametro BF deſcribatur



ſemicirculus ſecans AD in C. Tum ducta BC compleatur triangulum BAC. Nam $\frac{1}{4}$BAq+ADq=EFq.

Quare $\sqrt{q}: \frac{1}{4}BAq + ADq: \mp \frac{1}{2}BAq = \begin{cases} AF. \\ BF. \end{cases}$

Ducantur omnia in BA; fietque

$\sqrt{q}: \frac{1}{4}BAqq + (BAq*ADq)Mqq: \mp \frac{1}{2}BAq =$

$= \begin{cases} BA*AF = CAq. \\ BA*BF = BCq. \end{cases}$

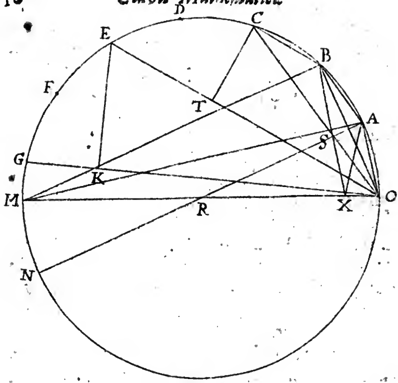
Con-

Conſectarium. Atque ex his duabus proportionibus patet æquationum, in quibus ſunt tres ſpecies æqualiter in ordine ſcalæ adſcendentes, quarum ſuprema ſit quadrato-quadratica, effectio Geometrica.

Probl: XXVI. De angulorum ſive peripheriarum biſectione, triſectione, quinquiſectione, ſeptiſectione, pauca etiam, ad Analytices præſtantiam, uſumque admirandum, oſtendendum, apponam, Geometricam quidem praxim adhuc inventam non habent: ſicut nec Meſolabium inventum eſt. At verò in Sectione 15 Cap. XVIII, Æquationes quaſdam Cubicas prælibavi; quas etiam ſolertia, aliàs innumeras Analytices ſtudioſus poterit comminiſci, quarum fortaſſe ope meſolabium hactenus tenebris obvolutum, in lucem tandem proferatur.

Diſtinguantur in peripheria ſeptem æquales partes ab O fine diametri literis A B C D E F G: ducantur ſubtenſæ, ſicut fit in ſchemate. Sumatur MX = MB. ducantur etiam AX & XB; & diameter NRA; & ad OE perpendicularis CT. Quoniam per 17, Cap: XVIII, Theor: 1, AB = AX: erunt triangula BMX, ORA, OAX, ſimilia; ideoque $\frac{OAq}{Rad} = OX$. Sunt etiam triangula OAB, ARM, ſimilia. Et per 47 e 1, MA $= \sqrt{q{:}4Radq - OAq}$.

His



His ſic præmiſſis, erit RA. MA, hoc eſt, Rad. $\sqrt{}$q : 4Radq — OAq :: OA. OB. Ergo $\frac{4Radq \times OAq - OAqq}{Radq} = OBq$: quæ eſt anguli duplicatio.

Et $4Radq \times OAq - OAqq = Radq \times OBq$: quæ eſt anguli biſectio.

Deinde quia OS=OA. & SA=OX. & NS=MX =MB. Erit per 17 c 18 Th:16, $\frac{NS \times SA}{OS} = SC$: hoc eſt $2Rad - \frac{OAq}{Rad}$ in $\frac{OAq}{Rad}$, diviſa per OA, vel

2Radq

$$\frac{2Radq \times OA - AOc}{Radq} = SC.$$ Et ſi addatur OA, fiet $$\frac{3Radq \times OA - OAc}{Radq} = OC:$$ quæ eſt anguli triplicatio. Et $3Radq \times OA - OAc = Radq \times OC$: quæ eſt anguli triſectio.

Item quia $2ET + CB = OE$ Et MO.MB::OC.OT: hoc eſt $2Rad. 2Rad. - \frac{OAq}{Rad} :: \frac{3Radq \times OA - OAc}{Rad}$ $$\frac{6Radqq \times OA - 5Radq \times OAc + OAqc}{2Radqq}:$$ E cujus duplo ſi tollatur OA: reſtabit $$\frac{5Radqq \times OA - 5Radq \times OAc + OAqc}{Rqq} = OE:$$ quæ eſt anguli quintuplatio. Et $OAqc - 5Radq \times OAc + 5Radqq \times OA = Radqq \times OE$: quæ eſt anguli quinquiſectio.

Atque hac forma progredi licet ad Sept. ſectionem inveniendam, Nempe $7Rcc \times OA - 14Rqq \times OAc + 7Rq \times OAqc + OAqqc = Rcc \times OG$.

Nam MO. MB :: OE. OK. Et $2OK - OC = OG$. Operationem ſtudioſis relinquo.

Verùm quia Radius ponitur 1, quæ in Multiplicatione & Diviſione, nihil mutat: idcircò in hiſce omnibus Æquationibus, Radius cum omnibus ſuis poteſtatibus, omitti poterit.

Sed quo artificio iſtiuſmodi operoſæ Æquationes (in quibus non ſunt tantùm tres ſpecies æqualiter in ordine

ordine ſcalæ adſcendentes) ſolvantur, quanquàm non eſt hujus inſtituti docere : tamen quid in hoc negotio in uſum nobiliſſimi doctiſſimique Domini Gerardi Aungier, Domini Aungier & Baronis de Longford, ante plurimos annos, commentus ſum, in gratiam ſtudioſorum Mathematices, qua poſſum brevitate, in lucem proferre non pigebit.

SOLI DEO GLORIA.

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DE ÆQVATIONVM ADFECTARUM RESOLUTIONE IN NUMERIS.

1. CONſtruendæ Æquationis adfectæ modus. Sumatur, ut lubet, pro B,3 : pro Cq,16 : pro Dc,125: pro Fqq,1296:&c. Nec refert utrum numeri ſumpti ſint verè figurati necne. Sitque ex his Coëfficientibus conſtruenda Æquatio Quadrato-cubica. Ea pro modo Tabulæ Analyticæ poſterioris in ordine Quadrato-cubico, conflata, eſto $Lqc - 5BLqq + 10CqLc - 10DcLq + 5FqqL = Gqc$. Quæ in numeris, ſtatuendo L (radicem) 47, erit $1qc - 15qq + 160c - 1250q + 6480l = 170304782$. vel omiſſa *unciarum* diſtinctione, pro 15qq, dic BLqq, pro 160c, dic CqLc; pro 1250q, dic DcLq; & pro 6480l, dic FqqL. Nam ſi L ſit 47; erit $Lq = 2209$: & $Lc = 103823$: & $Lqq = 4879681$: & $Lqc = 229345007$.

Conſtructionis hujus Practica.

BLqq	229345007
15×4879681	-73195215
CqLc	156149792
160×103823	$+16611680$
DcLq	172761472
1250×2209	-2761250
FqqL	170000222
6480×47	$+304560$
	170304782

2. Proponatur Æquatio quæcunque, puta modò inventam,

$1qc - 15qq + 160c - 1250q + 6480l = 170304782$:

Vel numeris in ſymbola mutatis,

$Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$:

Et ſi plures eſſent adfectionum Species, conſequenter efferri poterunt per Hcc, Kqqc, Mqcc, Nccc, & ſic ulterius.

3. Radicis L ex his inveſtigandæ duæ erunt partes, nempe A latus primum, & E latus ſecundum, ſive ſubſequens quodlibet. Quare $L = A + E$: & omnes poteſtates ex L, æquantur conſimilibus poteſtatibus ex $A + E$: v.g. $Lq = Aq + 2AE + Eq$: & $Lc = Ac + 3AqE + 3AEq + Ec$. &c.

Qui igitur numeroſam hanc poteſtatum adfectarum

rum reſolutionem cupit addiſcere, eum in purarum poteſtatum Geneſi & Analyſi, bene verſatum eſſe oportet.

4. In Æquatione propoſitâ, poteſtas reſolvenda 170304782, ſive Gqc, eſt Quadrato-cubica, cujus etiam generis ſunt ſingulæ adfectionum Species. Nam *Heterogenea inter ſe nec addi poſſunt, nec ſubtrahi.*

5. Quare in ſingulis adfectionibus duo ſunt conſideranda, Gradus adfectionis, & Coëfficiens: ut in 15 qq, adfectionis gradus eſt Quadrato-quadraticus, & coefficiens 15, lateralis: In 160 c, adfectionis Gradus eſt cubicus, & Coefficiens 160, Quadraticus: In 1250 q, adfectionis Gradus eſt Quadraticus, & coefficiens 1250, cubicus: denique in 6480 l, adfectionis gradus eſt lateralis, & coefficiens 6480, Quadrato-quadraticus: ſicut ex utraque Æquationis deſignatione comparata clariſſimè liquebit. Atque hinc duo oriuntur Conſectaria pro laterum ſingularium extractione.

6. Primum Conſectarium eſt, Si coefficientis pro ſua ſpecie, radix, ducta in adfectionis gradum, multiplicet ipſum coefficientem: factus erit ejuſdem generis cum poteſtate reſolvenda: Ut in præcedente Æquatione, ſi latus 15 Quadrato-quadraticè multiplicatum, ducatur in 15; & ſi √q 160 cubatum, ducatur in Quadratum 160; & ſi √c 1250 quadratum, ducatur in cubum 1250; denique ſi √qq 6480 ducatur in Quadrato-quadratum 6480; ex ſingulis hiſce multiplicationibus emerget numerus Quadrato-cubicus. *Atque hæc multiplicatio Analytica, modus*

eſt reducendi coefficientem quemlibet ad ſpeciem poteſtatis reſolvendæ, in lateris primi ſingularis extractione uſitatiſſimus.

7. Unde etiam clariſſimè liquet, quod ſi numerus ex coefficientibus hoc modo reductis, atque comparatis, emergens, minor ſit poteſtate reſolvenda, latus ipſius etiam minus eſt latere poteſtatis reſolvendæ; Si verò major, eſt majus; & ſi æqualis, æquale. In hac igitur Æquatione, 1qc-15qq+160c-1250q+6480l =170304782: vel 170304782+15qq-160c+1250q -6480l =1qc, ſi tum coefficiens lateralis 15, tum √q160, tum √c 125, tum √qq6480, Quadratocubentur; prodibunt quatuor adfectionum ſpecies homogeneæ, nempe 7593..,3238..,1450.., 0581.., Quod quidem per Logarithmos facillime fit, ſatiſque pro propoſito acuratè. Operationis ratio ex fine hujus Tractatus (ubi de Logarithmorum notitiâ pauca traduntur) petenda, ſic eſt.

Loga-

Logarithmi.	Numeri Coefficientes.
5 × 1,17609	15 qq
5,88045	7593..
2) 2,20412	160 c
5 × 1,10206	
5,51030	3238..
3) 3,09691	1250 q
5 × 1,03230	
5,16150	1450..
4) 3,81157	64801
5 × 0,95289	
4,76445	0581..

In Æ,quatione igitur propoſitâ,ſpeciebus pro ſignorum ratione in unam ſummam aggregatis, erit 170304700+759300—323800+145000—058100 =1qc=170827100. Quod etiam in aliis æquationibus ſimiliter fieri poterit.

8. Secundum eſt, Si poteſtas reſolvenda per Coefficientem dividatur, quotus ad ipſum adfectionis gradum referretur : hoc eſt, quotus erit latus, ſi adfectio ſit ſub latere; vel quadratum, ſi ſub quadrato ; & ſic de reliquis gradibus : Ut in priore Æquatione,

ſi 170304782 dividatur per 15, quotus erit Quadrato-quadraticus; ſi per 160, quotus erit cubicus; ſi per 1250, quotus erit Quadraticus; ſi denique per 6480, quotus erit lateralis. Quare non ſemper ipſe quotus, ſed ipſius plerumque radix pro adfectionis gradu, erit latus ſingulare eliciendum.

9. In ſecundæ etiam radicis inveſtigatione hoc teneri debet; quòd pro numero figurarum in quoto cenſendus fere erit ejus gradus: ut ſi quotus unicâ conſtet figura, ſit latus; Si duabus, Quadratum; ſi tribus, cubus, &c. Et ſi quotus ſuperet 5, vel 50, vel 500, &c. ad gradum fortaſſe ſequentem, in grandioribus præſertim adfectionibus, poterit extendi. Atque *hæ ſunt diviſionis Analyticæ leges.*

10. Nec in iſtiuſmodi *Multiplicatione* atque *Diviſione*, totam poteſtatem reſolvendam, cum toto Coefficiente, percurrere opus erit; Sed ſolummodo ad punctum congruens proximum.

11. Nam in reſolutione adfectarum Æquationum punctationes omnes graduum fieri debent, in poteſtate reſolvendâ, ſicut in puris: Supremi quidem gradus ſupra: reliquorum verò infra. Coefficientes etiam, pro ſuâ quiſque ſpecie, punctandi ſunt. Prioris exempli punctationes ſic erunt,

1qc − 15qq + 160c − 1250cq + 6480l = 170304782

12. Debet autem regulariter (præſertim ſi Coefficiens

ciens ſit negativus) numerus punctorum in ſingulis eſſe æqualis. Quare ſi poteſtas reſolvenda puncta plura, ſive pauciora habeat ſupra ſe, quam Coefficiens; tot deficienti præponantur circuli, ut puncta utrobique poſſint eſſe æqualia. Et in ſingulis lateribus eruendis punctum coefficientis lateri illi proprium, ad parile poteſtatis reſolvendæ punctum ſuperius, accommodandum eſt: quod quidem fiet, ſi unitatis locus in coefficiente, ad puncta poteſtatis inferiora gradui ſuo convenientia, ordine dimoveantur.

13. Si Coefficiens aliquis ſit fractio, ſive latus ſurdum; reducatur ad integros cum partibus decimalibus.

14. Et ſi opus ſit radicis eductionem in partibus decimalibus perſequi: poſt lineam ſeparatricem circulos quot viſum erit adſcribes, eoſque ſupra & ſubtus, punctis conſimiliter inſignire perges.

15. Tabula oſtendens tum *Diviſores*, tum *Gnomones*, pro laterum ſingularium in Æquationibus adfectis inveſtigatione; collecta & continuanda ex tabella Analytica poſteriore. Et nota, quod Coefficientis cujuſque ſpecies omnes ſunt adfirmatæ, ſi ipſa ſit adfirmata; negatæ vero, ſi negata.

Pro primo Latere.	Pro lateribus ſingularibus ſequentibus, ad complendum *Gnomonem*.
Aq BA	2AE . Eq BE . } = Cq
Ac BAq CqA	3AqE . 3AEq . Ec B2AE . BEq . CqE . } = Dc
Aqq BAc CqAq DcA	4AcE . 6AqEq . 4AEc . Eqq B3AqE . B3AEq . BEc . Cq2AE . CqEq . DcE . } = Fqq
Aqc BAq CqAc DcAq FqqA	5AqqE . 10AcEq . 10AqEc . 5AEqq . Eqc B4AcE . B6AqEq . B4AEc . BEqq . Cq3AqE . Cq3AE . CqEc . Dc2AE . DcEq . FqqE . &c. } = Gqc

16. *Diviſores* ubique ſumuntur ex iis, quæ in data habentur menſura, juſto ordine diſpoſitis, atque Aggregatis, habita ſignorum ratione.

17. Si Æquationis alicujus ſuprema poteſtas ſit negativa, Æquatio illa eſt ambigua.

18. Latus ſingulare primum elicitur ex his *Regulis*, deſumptis ex duobus conſectariis in Sect. 6. & 8.

Prima. Si Coefficiens ita longè in poſteriora decedit, ut vix ad primum poteſtatis reſolvendæ punctum pertingat; nec (Analyticè etiam reductus) enormem in illo mutationem faciat: in extractione lateris ſingularis primi, negligi omninò poterit.

Secunda. Si Coefficiens in anteriora prorumpit, ſitque adfirmativus: devolvendus eſt in puncta conſequentia, donec locus diviſioni fiat. Per quam diviſionem quotus inventus ad gradum adfectionis referretur. Quod etiam in extractione minoris radicis Æquationis ambiguæ intelligi debet.

Tertia. Si vero negativus ſit, & pluribus conſtet punctis, quam poteſtas reſolvenda; ſuppleantur loci deficientes circulis præfixis: & pro latere primo ſingulari, ſumatur ipſa coefficientis, pro ſuo genere, radix.

Quarta. Si utrobique puncta ſint æqualia, & numeri in primo tum coefficientis, tum poteſtatis reſolvendæ, puncto, non multum diſcrepent: Coefficiens per radicem ſuam, pro ſpecie qua punctatur, ſub congruente puncto extractam, ad poteſtatis ſpeciem (per Analyticam multiplicationem) reductus, poteſtati reſol-

reſolvendæ addatur, ſi ſit negativus; vel auferatur, ſi adfirmativus. Nam ſi ſit Ac+CqA=Dc, erit Ac =Dc∓CqA. At ſi Æquationis ambiguæ latus majus quæratur, Poteſtas reſolvenda è coefficiente reducto auferatur. Nam ſi ſit CqA-Ac=Dc, erit Ac=CqA -Dc. Tum ſummæ vel differentiæ radix, erit latus primum eliciendum. Et nota, quod Æquationis ambiguæ latus majus, aliquando per diviſionem; aliquando per extractionem radicis è coefficiente; ſed plerumquè per reductionem coefficientis inveſtigatur.

19. Atque his præceptis ſolerter perpenſis, Illud demùm verum latus ſingulare primum erit, quod primò omnium talem exhibet diagonalem, qui unà cum coefficientibus, ſicut Æquationis conditio poſtulat, juxta tabellam præcedentem, multiplicatis; omnibuſque in unam ſummam (diligente ubique tum ſignorum, tum ſedium reſpectu habito) aggregatis; numerum profert poteſtate reſolvendâ, unde ſubtrahendus eſt, non majorem. Notandum autem eſt, quòd numeris negativus quantuſcunque, minor eſt omni tum adfirmativo, tum negativo minore: ut -4 minor eſt quàm 1, & quam -1. Item quòd ſubductio mutat ſignum numeri ſubducendi: ut ex 4 tolle 6, reſtat 4-6, hoc eſt -2. Et ex -4 tolle -6, reſtat -4+6, hoc eſt 2. Denique ex 4 tolle -6, reſtat 4+6, hoc eſt 10. Quare in lateris primi ſingularis extractione, tentandum aliquoties eſt, donec latus verum inveneris; quod per proximè majus, certiſſimè agnoſces.

20. In

20. In conſtitutione diviſoris pro ſecundo latere inveſtigando; Coefficientis ductæ in gradum quemlibet, ſedes ordinari debet ſecundum proprii gradus punctationem: hoc eſt, Coefficientis ſub latere ſedes diſtabit verſus ſiniſtram, à puncto ſive ſede ipſius Coefficientis, uno loco: Coefficientis ſub quadrato ſedes, duobus locis: ſub cubo, tribus: &c. Et ob vitandam confuſionem, utile erit in reſiduo poteſtatis reſolvendæ, punctationes illas, quæ præſenti radici eruendæ inſerviunt, ſolas diſtinguere.

21. Tum latus ſingulare ſecundum elicietur ſic: Diviſores cujuſque generis, ex tabula præcedente inventi, & juſto ordine diſpoſiti, in unam ſummam aggregentur; & per totalem illum diviſorem, reliquum poteſtatis reſolvendæ dividatur. Nam quotus juxta diviſionis Analyticæ leges (ſi id uſus exigat) perpenſus, dabit latus ſingulare ſecundum eliciendum. Cæterùm in hac inveſtigatione multotiès, præſertim ſi magnitudinum dividentium negativarum aggregatum, aggregato adfirmativarum penè æquetur (adeò ut Diviſor Reliquo poteſtatis reſolvendæ minor admodum ſit) maxima ineſt lubricitas: quam tamen Analyſta ſagax facile effugiet.

22. Hæc igitur Regula eſto perpetua. Illud demùm verum latus ſingulare ſecundum eſt; quod primò omnium talem exhibet *Gnomonem*, conſtantem ex complementis cujuſque generis, & Coefficientibus, ſicut Æquationis conditio poſtulat, juxta tabulam præcedentem, multiplicatis; omnibuſque in unam ſummam,

ſummam, diligente ubique tum ſignorum, tum ſedium, habita ratione, aggregatis; qui *Gnomon* non major ſit poteſtate reſolvenda unde ſubtrahendus eſt. Quare tentandum aliquoties eſt, donec latus verum inveneris: quod etiam per proximè majus, certiſſime agnoſces.

23. Latera omnia ſingularia poſt ſecundum, per Diviſionem ſimplicem facillimè acquiruntur.

24. Si adfectiones ſint compoſitæ ex adfirmativis, & negativis: antecedentia præcepta mixtim ſunt cum ſolertia & judicio uſurpanda. Et in lateribus æſtimandis præponderabit ſemper adfectio major, minori. Verum totum hoc negotium Analyticum, quod verbis enarrare difficillimum foret, frequens exercitatio, tum in Geneſi, tum in Analyſi poteſtatum cujuſque generis, facile ſatis reddet, atque familiare.

25. Sed quia ſuperius aliquoties dictum eſt, tentatu opus eſſe;. quod quidem in adfectionibus multiplicibus, & ubi gradus ſunt elatiores, valde laborioſum erit: adponam hic, coronidis loco, duos modos ejuſmodi tentamenti levandi: unum per Depreſſionem, ex Cap. XVI. Sect. 9. *Clavis*: alterum per Canonem Logarithmorum 10000. In utroque autem ſi Æquatio fuerit ambigua, ſigna ejus omnia erunt mutanda. Notandum etiam hîc eſt, quod numerus negativus quantuſcunque, minor eſt omni tum adfirmativo, tum negativo minore.

26. Inventio laterum ſingularium per Depreſſionem. Si latus primum quæratur: In ſingulis Æquationis

tionis datæ ſpeciebus abſcindantur lineâ ſeparatrice omnia puncta poſt primum. Deinde adplicentur omnes ſpecies ad latus; hoc eſt, deprimantur uno gradu.

Exampl:I. 1qq-72c+2386001 = 8725815. Hæc Deprimendo fiet 1c+ 238,6-7,2q = L) 872,5.
Eſto A 4. erit 4) 872,5 (218,1, juſtus.
Et + 64 + 238,6 — 115,2 = 187,4, minor juſto.
Eſto A 5 erit 5) 872,5 (174,5, juſtus.
Et + 125 + 238,6 — 180,0 = 183,6; major juſto.
Latus igitur verum A = 5 - 1, hoc eſt, 4.

Exampl:II. De Æquatione ambigua. 1c-32571 = -45744. Hæc deprimendo fiet 1q-32,5 = L) -45,7.
Eſto A 4. Erit 4) — 45,7 (— 11,4, juſtus.
Et + 16 — 32,5 = -16,5, minor juſto.
Eſto A 5. erit 5) — 45,7 (— 9,1, juſtus.
Et + 25 — 32,5 = — 7,5, major juſto.
Latus igitur verum A = 5 — 1, hoc eſt, 4.

Si latus ſecundum quæratur: In ſingulis ſpeciebus abſcindantur omnia puncta poſt ſecundum. Deinde adplicentur omnes ſpecies ad quadratum; hoc eſt, deprimantur duobus gradibus. Ut in Exemplo I.

1qq — 72c + 2386001 = 8725815. Hæc Deprimendo fiet 1q+L) 238600-72.l = Q) 8725815.

Eſto A 47: erit 2209) 8725815 (3949. Juſtus.
Et 2209+5077—3384=3896. minor juſto.
Eſto A 48: erit 2304) 8725815 (3787. Juſtus.
Et 2304+4971-3456=3819: major juſto.
Latus igitur verum eſt 48— 1, hoc eſt, 47.

27. Inventio lateris ſingularis ſecundi per *Logarithmos.*

Index Logarithmi cujuſque deſumitur ex tabella in initio *Clav:* pro diſtantia primæ ſuæ figuræ, ante vel poſt locum unitatum, cujus Index eſt 0. Eædem igitur figuræ, eodem ordine diſpoſitæ, eundem habeat Logarithmum: Indices vero diverſi eſſe poſſunt. Ut numeri 436, Log: eſt 2,6394865 at numeri 43600, eſt 4,6394865. & numeri 4,36, Log: eſt 0,6394865. Denique numeri 0,00436, Log: eſt $\bar{3}$,6394865.

Summæ duorum Logarithmorum, Logarithmus eſt facti à valoribus: differentiæ autem, Logarithmus eſt quoti. Ut quia 4,36×9=39,24 hujus Log: 1,5937290=0,6394865 + 0,9542425. Et quia 9) 39,24(4,36: hujus Log: 0,6394865 = 1,5937290 —0,9542425.

Logarithmus lateris, ductus in numerum dimenſionum cujuſque poteſtatis, eſt ejuſdem poteſtatis Logarithmus:

rithmus : Ut quia numeri 436, Log : eſt 2,6394865 : Erit 2,6394865×2 = Log: Q:436. Et 2,6394865×3 = C: 436: Et 2,6394865×4 = QQ:436. &c.

Logarithmus poteſtatis cujuſque diviſus per numerum dimenſionum ſuarum, exhibet Logarithmum radicis ſuæ.

Si in Serie Geometricè continuè proportionalium Logarithmus primi termini tollatur è Logarithmo ſecundi, reliquus erit Logarithmus rationis : Qui, ſi in numerum terminorum minus uno (qui numerus eſt rationum) ducatur; deindeque Logarithmo primi termini augeatur; Logarithmus erit termini ultimi.

28. Atque hæc de Logarithmorum notitia ſatis ſunto : quibus intellectis, reliquam operationem, exempla ſequentia diligenter inſpecta, facilem reddent: In qua etiam omnes punctationes, poſt duas primas, linea ſeparatrice abſcindendæ ſunt.

Exampl:I. 1qq−72c+238600l = 8725815. Juſtus. Sunto duo prima latera ſingularia.

47·	1,6720 9 | 8	−72	+238600
		1,85733	5,37767
Cu:	5,01629	5,01629	1,67210
QQ:	6,68839	6,87362	7,04977
	+4880	−7475	+11213

Et +4880+11213−7475 = 8618: minor juſto.

48. 1,68124|1 1,85733 5,37767
Cu. 5,04372| 5,04372 1,68124
QQ.6,72496| 6,90105 7,05891
∴ +5308 —7963 + 11455
Et +5308+11455-7963 = +8800 : major juſto.
Radix igitur vera erit 48 - 1, hoc eſt, 47.

Eampl:II. 1c—3257l = —45744. Juſtus.
Sunto duo prima latera ſingularia.

|—3257|
48. 1,68124|1 3,51282
Cu. 5,04372| 1,68124
+1106 5,19406
-1563.

+1106—1563 = —457, minor juſtus.

49. 1,69019 6 3,51282
Cu. 5,07059| 1,69020
+1176 5,20302
—1596

+1176—1596 = —420, major juſto.
Radix igitur vera erit 49 — 1, hoc eſt, 48.

Latus ſecundum inveſtigari poterit per Logarithmos, etiam Depreſſione præcedente. Ut in Exemplo V. 1qq —1246 00q = 08972 6256. Hæc quadraticè depreſſa fiet 1q —1246 = Q) 8972,6.

Supponantur duo prima latera ſingularia,

,8972,6,

34. 1,53148 | 3,95337
Q. 3,06296 | 3,06296
+ 1156 | 0,89041: valor 7,77 Juſtus.

+ 1156—1246 = −90: minor juſto.

36. 1,55630 | 3,95337
3,11260 | 3,11260
+ 1296 | 0,84077: valor 6,93 Juſtus.

+ 1296—1246 = +50: major juſto.

Radix igitur vera cadit inter 34 & 36.

Atque hoc modò in XXVIII Sectionibus, ſive Præceptis (qui numerus eſt perfectus) doctrinam de Æquationum adfectarum reſolutione in numeris, adjuvante Deo omnium bonorum Datore, expedivi: Ejus igitur ſit omnis laus, honor, & gloria in ſempiternum. Amen.

Exempla quædam Æquationum Reſolutarum in Numeris.

EQuationum Quadraticarum, omniumque in quibus ſunt tres ſpecies in ordine ſcalæ æqualiter adſcendentes, Analyſi ſuperſedebo : quia in Cap: XVI. Sect 9. *Clavis*, modus facilior traditus eſt, quàm per generalem hanc methodum præſtari poterit : Et ad Exempla Æquationum aliter adfectarum progrediar. Denique in fine, Notas ad Exempla, ſubjungam ; in quibus operationis ratio, in laterum ſingularium inveſtigatione, ex præceptis ſuperiùs traditis, aperietur.

Initium faciam à Reſolutione numeroſæ Æquationis primò conſtitutæ, Nempe

1qc−15qq+160c−1250q+6480l = 170304782.
Hoc eſt, Lqc−BLqq+CqLc−DcLq+FqqL = Gqc

Examplum

Exemplum I.

1qc-15qq+160c—1250q+06480l = 170304782.
Hoc eſt, Lqc-BLqq+CqLc-DcLq+FqqL = Gqc.

	1703047 82	(47
	15	—B
	1250	—Dc
	160	Cq
	6480	Fqq
	1024	Aqc
	10240	CqAc
	25920	FqqA
+	11289920	
	3840	—BAqq
	20000	—DcAq
−	404000	
	7249920	*Ablatit.*
R	9780558 2	
	1280	5Aqq
	640	10Ac
	160	10Aq
	20	5A
	7680	Cq3Aq
	1920	Cq3A
	160	Cq
	6480	Fqq

Exempla quædam Æquationum Reſolutarum in Numeris.

EQuationum Quadraticarum, omniumque in quibus ſunt tres ſpecies in ordine ſcalæ æqualiter adſcendentes, Analyſi ſuperſedebo : quia in Cap: XVI. Sect 9. *Clavis*, modus facilior traditus eſt, quàm per generalem hanc methodum præſtari poterit : Et ad Exempla Æquationum aliter adfectarum progrediar. Denique in fine, Notas ad Exempla, ſubjungam ; in quibus operationis ratio , in laterum ſingularium inveſtigatione , ex præceptis ſuperiùs traditis , aperietur.

Initium faciam à Reſolutione numeroſæ Æquationis primò conſtitutæ, Nempe

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Examplum

Exemplum I.

1qc-15qq+160c-1250q+6480l = 170304782.
Hoc eſt, Lqc-BLqq+CqLc-DcLq+FqqL = Gqc.

1703047 82	(47
15	—B
1250	—Dc
160	Cq
6480	Fqq
1024	Aqc
10240	CqAc
25920	FqqA
+ 11289920	
3840	—BAqq
20000	—DcAq
— 404000	
7249920	*Ablatit.*
R 97805582	
1280	5Aqq
640	10 Ac
160	10 Aq
20	5 A
7680	Cq3Aq
1920	Cq3A
160	Cq
6480	Fqq

+142	50040	
38	40	- B4Ac
1	440	- B6Aq
	240	- B4A
	15	- B
1	0000	- Dc2A
	1250	- Dc
-40	87665	
+101	62375	*Diviſor.*
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	- B4AcE
70	560	- B6AqEq
8	2320	- B4AEc
	36015	- BEqq
7	0000	- Dc2AE
	61250	- DcEq
-355	56465	
+978	05582	*Ablatit.*

Exam.

Examplum II

1c + 420000 l = 247651713

Hoc eſt, Lc+CqL = Dc.

	247|651|713	(417
	42|0000	Cq
	64	Ac
	168|0000	CqA
	232|0000	*Ablatit.*
Rc	15|651|713	
	4|8	3Aq
	|12	3A
	4|20000	Cq
	9|12000	*Diviſor.*
	4|8	3AqE
	|12	3AEq
	1	Ec
	4|20000	CqE
	9|121|00	*Ablatit.*
Rc	6|530|713	
	|504|3	3Aq
	1|23	3A
	|420|000	Cq
	|925|530	*Diviſor.*
	3|530|1	3AqE
	6027	3AEq
	343	Ec
	2|940|000	CqE
	6|530|713	*Ablatit.*

4		1
16	8	
		1
16	8	1

Exem-

Exemplum III.

$1c + 1007q = 247617936,$

Hoc eſt, $Lc + BLq = Dc.$

247	617	936	(417
10	07		B
64			Ac
161	12		BAq
225	12		*Ablatit.*
℞ 22	497	936	
4	8		3Aq
	12		3A
8	056		B2A
	100	7	B
13	076	7	*Diviſor.*
4	8		3AqE
	12		3AEq
	1		Ec
8	056		B2AE
	100	7	BEq
13	077	7	*Ablatit.*

Rc	9	4	2	0	2	3	6		4	1
		5	0	4	3			3Aq	16	
				1	2	3		3A		8
		8	2	5	7	4		2BA		1
				1	0	0	7	B	16	81
	1	3	3	2	2	7	7	*Diviſor.*		
	3	5	3	0	1			3AqE		
			6	0	2	7		3AEq		
					3	4	3	Ec		
	5	7	8	0	1	8		B2AE		
			4	9	3	4	3	BEq		
	9	4	2	0	2	3	6	*Ablatit.*		

1qq--442990051=22252086

Lqq—DcL=Fqq.

	0 2 2 2 5	2 0 8 6		(345
--4 4	2 9 9 0	0 5		—Dc
+8 1				Aqq
--1 3 2	8 9 7 0	1 5		—DcA
—5 1	8 9 7 0	1 5		*Ablatit*
℞ 5 2	1 1 9 5	3 5 8 6		
1 0	8			4Ac
	54			6Aq.
	12			4A
+1 1	3 5 2			
--4	4 2 9 9	0 0 5		—Dc
+6	9 2 2 0	9 9 5		*Diviſor.*
5 4	0			4AcE
1 3	5 0			6AqEq
1	5 0 0			4AE
	6 2 5			Eqq
+6 9	0 6 2 5			
--2 2	1 4 9 5	0 2 5		—DcE
+4 6	9 1 2 9	9 7 5		*Ablatit.*
℞ 5	2 0 6 5	3 8 3 6		
+ 1	2 7 9 3	7 3 9 5		*Diviſor*
+ 5	2 0 6 5	3 8 3 6		*Ablatit.*

1qq-124600q=08972625 6,Lqq-CqLq=Fqq.

08972625 6	(354
-124600	-Cq
+81	Aqq
-1121400	-CqAq
-311400	*Ablatit*
R 32037262 56	
108	4Ac
54	6Aq
12	4A
+11352	
747600	-Cq2A
124600	-Cq
-7600600	
+3751400	Diviſor
540	4AcE
1350	6AqEq
1500	4AEc
625	Eqq
+690625	
3738000	-Cq2AE
3150000	-CqEq
-40495000	
+28567500	*Ablatit*
R34697625 6	
848918 00	*Diviſor*
† 346976256	*Ablatit*

Exem-

Exemplum VI.

1qq-340c=621066096
Lqq-BLc=Fqq.

6	2 1 0 6	6 0 9 6	(354
-3	4 0		-B
+8 1			Aqq
--9 1	8 0		BAc
-1 0	8 0		*Ablatis*
℞ 1 7	0 1 0 6	6 0 9 6	
1 0	8		4Ac
	5 4		6Aq
	1 2		4A
+1 1	3 5 2		
9	1 8 0		-B3Aq
	3 0 6	0	-B3A
	3 4	0	-B
--9	4 8 9 4	0	
+1	8 6 2 6	0	*Diviſor*
5	4	0	4AcE
1 3	5 0		6AqEq
1	5 0 0		4AEc
	6 2 5		Eqq

+6

+690625		
45900		-B3AqE
76500		-B3AEq
42500		-BEc
-5397500		
+1508750		*Ablatit*
℞ 1923 16096		
+ 46929060		Diviſor
+ 1923 16096		*Ablatit*

Exem. VII.

Exemplum VII.

1qq—771080001=085530576 Lqq-DcL=Eqq.

085530576	(426		
−77108000	—Dc		
+256	Aqq		
−308432000	—DcA		
−52432000	*Ablatit*		
Rc 532873 0576		4 · 2	
25.6	4Ac	16	
96	6Aq	16	
16	4A	4	
+26576		1764	
−77108000	—Dc		
+188652000	*Diviſor*		
512	4AcE	4 · · 2	
384	6AqEq	1	
128	4AEc	·646 8	
16	Eqq	948	
+551696		74088.	
−15421 6000	—DcE		
+3974800000	*Ablatit*		
Rc 135393 0576			
+2203040800	*Diviſor*		
+135393 0576	*Ablatit.*		

Exemplum

Exemplum VIII.

$32001 - 1c = 46577$ Æquatio eſt
CqL — Lc = Dc ambigua.

(47 *Radix major.*

4 6 5 7 7	
3 2 0 0	Cq
-6 4	-Ac
+1 2 8 0 0	CqA
+6 4 0 0	*Ablatit.*
R-1 7 4 2 3	
4 8	-3Aq
1 2	-3A
-4 9 2	
+3 2 0 0	Cq
-1 7 2 0	*Diviſor.*
3 3 6	-3AqE
5 8 8	-3AEq
3 4 3	-Ec
-3 9 8 2 3	
+2 2 4 0 0	CqE
-1 7 4 2 3	*Ablatit.*
R 0 0 0 0 0	

$$3200l - 1c = 46577$$

(157 *Radix minor.*

46	577	
32	00	Cq
–1		Ac
+32	00	CqA
31	00	*Ablatit.*
℞ 15	577	
	3	2Aq
	3	3A
–	33	
+3	200	Cq
2	870	*Diviſor.*
1	5	-3AqE
	75	-3AEq
	125	-Ec
–2	375	
+16	000	CqE
13	625	*Ablatit.*

℞ 1	952	000	
	252	05	*Diviſor.*
	745	107	*Ablatit.*
℞ 6	206	893	000 &c.

Exem. X.

53q−1c = 13254 Æquatio eſt ambigua.
BLq−Lc=Dc

13	2 5 4	(49	*Radix major.*
5	3	B	
−64		-Ac	
+84	8	BAq	
+20	8	*Ablatit*	
℞ −7	5 4 6		
4	8	-3Aq	
	1 2	-3A	
−4	9 2		
4	2 4	B2A	
	5 3	B	
+4	2 9 3		
−	6 2 7	*Diviſor*	
33	6	-3AqE	
5	8 8	-3AEq	
	3 4 3	-Ec	
−39	8 2 3		
29	6 8	B2AE	
2	5 9 5	BEq	
+32	2 7 7		
−7	5 4 6	*Ablatit*	
℞ 0	0 0 0		

Exemplum XI.

$$539q - 1c = 13254$$

13254	(20,05	*Radix minor.*
53	B	
-8	-Ac	
212	BAq	
132	*Ablatit*	

℞	54000000	
	-- 120000	--3Aq
	-- 600	--3A
	- 1200600	
	21200	B2A
	53	B
	+ 212053	
	919930	*Diviſor*
	- 6000000	--3AqE
	-- 15000	--3AEq
	-- 125	--Ec
	-- 60150125	
	106000	B2AE
	1325	BEq
+	1061325	
	45982375	*Ablatit*
℞	9017625000	&c

Exemplum

Exemplum XII.

600341 — 1c = 1023768

CqL — Lc = Dc.

1 0 2 3 7 6 8	(2 3 6	Radix major.
6 0 0 3 4	Cq	
—8	-Ac	
+ 1 2 0 0 6 8	CqA	
+ 4 0 0 6 8	*Ablatit.*	
R—2 9 8 3 0 3 2		
1 2	-3Aq	
6	-3A	
— 1 2 6		
+ 6 0 0 3 4	Cq	
— 6 5 9 6 6	*Diviſor.*	
3 6	-3AqE	
5 4	-3AEq	
2 7	-Ec	
— 4 1 6 7		
+ 1 8 0 1 0 2	CqE	
— 2 3 6 5 9 8	*Ablatit.*	
R — 6 1 7 0 5 2		
— 9 6 3 6 6	*Diviſor.*	
6 1 7 0 5 2	*Ablatit.*	
L		

Exemplum XIII.

$600341 - 1c = 1023768$

17,135 Radix minor.

1 0 2 3 7 6 8	
6 0 0 3 4	Cq
− 1	− Ac
+ 6 0 0 3 4	CqA
+ 5 9 9 3 4	*Ablatit*
℞ 4 2 4 4 2 8	
3	− 3Aq
3	− 3A
− 3 3	
+ 6 0 0 3 4	Cq
5 9 7 0 4	*Diviſor*
2 1	− 3AqE
1 4 7	−− 3AEq
3 4 3	− Ec
− 3 9 1 3	
+ 4 2 0 2 3 8	CqE
+ 4 1 6 3 2 5	*Ablatite*
℞ + 8 1 0 3 0 0 0	
5 9 1 6 1 9	*Diviſor.*
6 0 1 6 1 8 9	*Ablatit.*

```
R 2|0 8 6|8 1 1|0 0 0|
   |5 9 1|5 6 2|5 7   |Diviſor
 1 7 7 5|5 5 6|9 0 3  |Ablatit

 R|3 1 1|2 5 4|0 9 7|0 0 0|
  |  5 9|1 5 3|6 3 7 9 1  |Diviſor.
  |2 9 5|7 6 7|1 6 1 6 2 5|Ablatit

  |R 1 5|4 8 6|9 3 6|3 7 5|0 0 0|&c.
```

Exemplum XIV.

1qq--72c+238600l = 87258157056

• Lqq--BLc+DcL=Fqq.

```
8 7 2|5 8 1 5|7 0 5 6  (47,6
     --7|2      |--B
+ 2 3 8|6 0 0   |Dc
  2 5 6|        |Aqq
  9 5 4|4 0 0   |DcA
+ 1 2 1 0|4 0 0 |
  - 4 0 6|8     |BAc
+ 7 4 9|6 0 0   |Ablatit.
```


R 1 2 2

Exem. XVI.

℞ 1 2 2 | 9 8 1 5 | 7 0 5 6	
2 5 | 6	4Ac
9 6	6Aq
1 6	4A
2 3 | 8 6 0 0	Dc
+ 5 0 | 4 3 6 0	
3 4 | 5 6	--B3Aq
8 6 4	--B3A
7 2	--B
− 3 5 | 4 3 1 2	
+ 1 5 | 0 0 4 8	*Diviſor.*
1 7 9 | 2	4AcE
4 7 | 0 4	6AqEq
5 | 4 8 8	4AEc
2 4 0 1	Eqq
1 6 7 | 0 2 0 0	DcE
+ 3 9 8 | 9 8 8 1	
2 4 1 9 2	-B3AqE
4 2 | 3 3 6	-B3AEq
2 | 4 6 9 6	-BEc
− 2 8 6 | 7 2 5 6	
1 1 2 | 2 6 2 5	*Ablatit.*
℞ 1 0 | 7 1 9 0 | 7 0 5 6	
1 | 7 6 9 8 | 8 0 8	*Diviſor.*
1 0 | 7 1 9 0 | 7 0 5 6	*Ablatit*

Exem. XV.

31-1c=1,258640782100 CqL-Lc=Dc

	125	86 40	7 8 2	1	(0,4499 Subtenſa Gr: 26.
	33	Cq			
	-6 4	-Ac			
+	12	CqA			
	1136.	*Ablatit.*			
Rc	1 2 2	6 4 0			
	4	8	-3Aq		
		1 2	-3A		
	-4	9 2			
	+ 3		Cq		
	2 5	0 8	*Diviſor*		
	1 9	2	-3AqE		
	1	9 2	-3AEq		
		6 4	-Ec		
	-2 1	1 8 4			
+	1 2		CqE		
	9 8	8 1 6	*Ablatit*		
Rc	2 3	8 2 4	7 8 2		
	2	4 17	8 8	*Diviſor.*	
	2 1	6 6 5	1 5 1	*Ablatit.*	
Rc	2	1 5 9	6 3 1	1 0 0	
	.	2 3 9	5 0 6	2 3	*Diviſor.*
	2	1 5 4	5 8 5	5 0 1	*Ablatit.*
Rc		0 5	0 4 5	5 9 9	0 0 0

1qc-5c+5l = 1,147152872702092

Lqc-CqLc+FqqL = Gqc.

1	14715	28727	02092	(0,2437 Subtenſa *Gr*:14.
+	5	Fqq		
	-5	-Cq		
	32	Aqc		
1	0	FqqA		
+ 1	00032			
	—40	CqAc		
	96032	*Ablatit.*		
℞	18683	28727		
	8	0	5Aqq	
		80	10Ac	
		40	10Aq	
		10	5A	
	5		Fqq	
	+50088	410		
	60		-Cq3Aq	
	30		-Cq3A	
		5	-Cq	
	—630	5		
	+43783	410	*Diviſor.*	
	32	0	5AqE	
	4	80	10AEq	
	2	560	10AqEc	
		2560	5AEqq	
		1024	Eqc	
	20		FqqE	
+	20039	62624		

	240 480 320		-Cq3Aq -Cq3AE CqEc	E q
	-29120			
	17127	62624	*Ablatit.*	
℞	1555	66103	02092	
	4149122			*Diviſor.*
	124265012094 43			*Ablatit.*
℞	313	01090	92649	00000

Notæ in Exempla præcedentia.

IN Exemplis Sectionum 26 & 28, Numerum Juſtum voco eum, qui oritur ex adplicatione poteſtatis Reſolvendæ ad gradum lateris ſuppoſiti, per quem facta eſt Depreſſio. Hæc enim menſura eſt, cui reliquæ ſpecies omnes legitimè aggregatæ, deberent eſſe æquales. Ut in Exemplo 1° Sectionis 26, 1 c + 238,6 — 7,2 q = L) 872,5. Si pro latere primo ſupponatur 5 : Oportet eſſe C:5: + 238,6 — 7,2 Q:5: = 872,5 diviſum per 5: hoc eſt 125 + 238,6 — (7,2 × 25) 180, nempe 183,6 æqualem eſſe 174,5 Juſto. At major eſt: ideoque latus verum minus eſt quàm 5. Supponatur igitur iterùm 4 : Et periculum fac, an C:4: + 238,6 — 7,2 Q:4: æquetur 872,5 diviſo per 4.

Cæterùm nè in his Exemplis, ſicut etiam in ſequentibus, tentamenta hæc caſu merè fortuito ſuſcipiantur; Monendum erit.

Primò ſi lateris eruti homogenea poteſtas excedat poteſtatem Reſolvendam: vel ſi magnitudines augentes poteſtatem Reſolvendam, excedant eas quæ imminuunt : Latus A verum minus (ut plurimùm) erit latere eruto : Sin aliter, majus. Ut in hac Æquatione,

1 c + 260000 l = 1 8 0 9 3 1 7 1 3.

Secundò, Si Diviſores ſub eodem ſigno cum Reliquo

quo poteſtatis Reſolvendæ, excedant eos, qui ſunt ſub ſigno diverſo: Latus E verum (ut plurimum) minus erit quam Quotus: ſin aliter, majus: Ut in hac Æquatione, 15681 — 1c = 21952. Idem etiam accidit in Æquationibus ambiguis, quando Reliquum poteſtatis Reſolvendæ eſt adfirmativum: ut in hac Æquatione, 67681 — 1c = 214273. Harum trium Æquationum ſolutio in praxi, poſt Notas oſtendetur.

Tertio, ſi poſt hæc Monita, nihilominus ſubſit dubitatio; tentamentum à 5 commodiſſimè erit inchoandum: Atque inde per numeros impares continuanda inquiſitio: ſive ea per Depreſſionem fiat, ſive per Logarithmos.

His præmonitis, reſtat ut Exempla ipſa diſcutiamus.

Ad Exempl: I. √qc1703 eſt 4+, per Sect: 18, Reg: 1. Nam ut ex Sect: 7. adparet, per Coefficientes Analyticè reductos, non fit in primo puncto notabilis immutatio. Quare latus A verum erit 4.

Latus E verum mius eſt quàm Quotus 9: quia Diviſores ſub ſigno + (quod ſignum eſt ipſius ℞) excedunt eos qui ſunt ſub ſigno —.

Ad Exempl: II. 42)247(6—, per Sect: 18, Reg: 2. Nam 42 Analyticè reductus, per Sect: 6 & 8, fit 252: major quàm 247. Eſtque Latus A verum minus qnàm 6; quia C:6—: excedit 247.6.

Ad Exempl: III. 10) 247 (24+Q:5—: per Sect: 18, Reg: 2.

Ad

Ad Exempl: IV. √c44|3 eſt 3+, per Sect: 18, Reg:3. Quare latus A verum eſt 3.

Latus E verum minus eſt quàm Quotus 8—, per Monit:2.

Ad Exempl: V. √q12|4 eſt 3+, per Sect: 18, Reg:3. Quare latus A verum eſt 3.

Latus E verum minus eſt quàm Quotus 9—, per Monit:2.

Ad Exempl: VI. Coefficiens lateralis 3|4 Quadrato-quadraticè multiplicatus, & auctus 6|2, fit 140, QQ:3+: per Sect: 18, Reg: 4. Quare latus A verũ eſt 3.

Latus E verum minus eſt quam Quotus 9—, per Monit:2.

Ad Exempl: VII. √c77 eſt 4, per Sect: 18, Reg:3. Quare latus A verum eſt 4.

Ad Exempl: VIII. √q32 eſt 5|65, in 32 ſit 180|8 mi 46|5, reſtat 144, C:5: per Sect: 18, Reg:4. At 144 excedit 46|5. Quare Latus A verum minus eſt quàm 5, per Monit: 1.

Latus E verum minus eſt quam Quotus 10, per Monit: 2.

Ad Exempl: IX, XI, XIII. Solutio facillima eſt per Diviſionem, juxta Sect: 18, Reg:3.

Ad Exempl: X. C:5: eſt 125, mi 13, reſtat 112, C:5—: per Sect: 18, Reg:4, At 112 excedit 13. Quare latus A verum minus eſt quam 5, per Monit: 1.

Latus E verum minus eſt quàm Quotus 12, per Monit: 2.

Ad

Ad Exempl:XII. $\sqrt{q}6$ eſt 2 +, in 6 fit 12, mi 1, reſtat 11, C:25 per Sect: 18, Reg:4. At 11 excedit 1. Quare latus A verum paulò minus quam 2 +, per Monit.1.

Latus E verum minus eſt quàm Quotus 5 −, per Monit: 2.

Ad Exempl: XIV. QQ:7|2: eſt — 2687. Et $\sqrt{c}$238|6 eſt 6|2, cujus QQ eſt + 1480. Tum — 2687 + 1480 = — 1207: Hic additus ad 872, dat 2079, QQ:6 +: per Sect:18, Reg:4. Et quia adjectitius — 2687 major eſt quam ablatitius + 1480, erit latus A verum minus quàm 6, per Monit: 1.

Latus E verum minus eſt quam Quotus 9, per Monit: 2.

Ad Exempl: XV, XVI, Quia in utroque Æquationis ambiguæ Radix minor quæritur, nec obſtant Coefficientes etiam reducti, Analyſis per Divis: fiet, juxta Sect: 18, Reg: 1.

Praxis Exempli in Monito primo.

1c + 26|0000 = 180931713.

180|9 (4, latus A
26|0 Cq.

$\sqrt{q}26$ eſt 5, in 26 fit 130, tollatur ex 180, reſtat 50, C:3 +: qui minor eſt quam 180. Quare latus A verum majus eſt quàm 3.

Praxis

Praxis Exempli in Monito ſecundo.

1568l − 1c = 21952

21	952	(28, Duo prima latera.
15	68	Cq
−8		Ac
+31	63	CqA
+23	36	*Ablat*
R − 1	408	
1	2	−3Aq
	6	−3A
−1	26	
+1	568	Cq
+	308	*Diviſor*

Signum R eſt −. At − 1,26 minor eſt quam + 1,568. Quare latus E verum majus eſt quam Quotus 4.

Praxis Exempli poſterioris in Monito ſecundo.

$$67681 - 1c = 214273$$

2 1 4	2 7 3	(47,	Duo prima latera.
6 7	6 8	Cq	
− 6 4		−Ac	
+ 270	7 2	CqA	
+ 206	7 2	*Ablat*	
R + 7	5 5 3		
4	8	− 3Aq	
	1 2	− 3A	
− 4	9 2		
+ 6	7 6 8	Cq	
+ 1	8 4 8	*Diviſor.*	

Signum R̸ eſt +. At Diviſor ex A lateris gradibus negativus, minor eſt Diviſore Coefficiente adfirmativo; hoc eſt −4,92 minor eſt quam +6,768. Quare latus E verum majus erit quam Quotus 4.

In Sectione XXVII. Logarithmorum doctrinam paucis tradidi: Sed ſatis luculentèr præſertim pro tribus prioribus Numerationis ſpeciebus, ſcilicet Additione, Subductione, & Multiplicatione.

Operatio

Operatio quidem in Addendo & Subtrahendo, ſi Indices ſint adfirmativi, à communi integrorum viâ nihil differt: parum etiam ſi ſint negativi, ut ex his Exemplis apparet,

Inventio fractionum	13. 1,11394. / 17. 1,23045.	Et	15. 1,17609 / 32. 1,50515
	Log: $\bar{1}$,88349		Log: $\bar{1}$,67194

Additio.		Subductio.	
Ad	$\bar{1}$, 88349	Ex	$\bar{1}$, 88349
adde	$\bar{1}$, 67194	tolle	$\bar{1}$, 67194
Sum	$\bar{1}$, 55543	Reſt	0, 21155

Multiplicatio.

Lateris	0,0064	Lateris	0,0064
3 ×	$\bar{3}$, 80614	2 ×	$\bar{3}$, 80614
Cubus	$\bar{7}$, 41842	Quadr:	$\bar{5}$, 61228

Diviſionis Logarithmi Indicem habentis negativum, per 2,3,4,5, &c. difficultas conſtat in inveſtigatione Indicis Quoti. Cui rei hæc inſervit Tabella.

Diviſores

Diviſores.	2)	$\bar{1} . \bar{2}$	$\bar{1}$
		$\bar{3} . \bar{4}$	$\bar{2}$
		$\bar{5} . \bar{6}$	$\bar{3}$
		$\bar{7} . \bar{8}$	$\bar{4}$
	3)	$\bar{1} . \bar{2} . \bar{3}$	$\bar{1}$
&c		$\bar{4} . \bar{5} . \bar{6}$	$\bar{2}$
		$\bar{7} . \bar{8} . \bar{9}$	$\bar{3}$
	4)	$\bar{1} . \bar{2} . \bar{3} . \bar{4}$	$\bar{1}$
		$\bar{5} . \bar{6} . \bar{7} . \bar{8}$	$\bar{2}$
	5)	$\bar{1} . \bar{2} . \bar{3} . \bar{4} . \bar{5}$	$\bar{1}$
		$\bar{6}\ \bar{7}\ \bar{8}\ \bar{9}\ \overline{10}$	$\bar{2}$
		40 30 20 10 0	

In hac Tabella Diviſores ſunt à ſiniſtrâ intra lineam flexam.

Tum verùs dextram ſequuntur Logarithmorum dividendorum Indices negativi.

His in ſingulis ordinibus collaterales adſtant Quotorum Indices etiam negativi.

Subtùs autem qui ſcribuntur numeri, 0, 10, 20, 30, 40, Oſtendunt numeros addendos primæ figuræ Logarithmi dividendi, cujus Index negativus invenitur ſupra in eâdem columnâ, juxta Diviſorem. Ut ſi Logarithmus

Operatio quidem in Addendo & Subtrahendo, ſi Indices ſint adfirmativi, à communi integrorum viâ nihil differt: parum etiam ſi ſint negativi, ut ex his Exemplis apparet,

Inventio fractionum	$\frac{13.\ 1,11394}{17.\ 1,23045}$	Et	$\frac{15.\ 1,17609}{32.\ 1,50515}$
	Log: $\bar{1}$,88349		Log: $\bar{1}$,67194

Additio.		Subductio.	
Ad	$\bar{1}$, 88349	Ex	$\bar{1}$, 88349
adde	$\bar{1}$, 67194	tolle	$\bar{1}$, 67194
Sum	$\bar{1}$, 55543	Reſt	0, 21155

Multiplicatio.

Lateris 0,0064	Lateris 0,0064
3 * $\bar{3}$, 80614	2 * $\bar{3}$, 80614
Cubus $\bar{7}$, 41842	Quadr: $\bar{5}$, 61228

Diviſionis Logarithmi Indicem habentis negativum, per 2,3,4,5, &c. difficultas conſtat in inveſtigatione Indicis Quoti. Cui rei hæc inſervit Tabella.

Diviſores

	Diviſor						Quot.
Diviſores.	2)				$\bar{1}$	$\bar{2}$	$\bar{1}$
					$\bar{3}$	$\bar{4}$	$\bar{2}$
					$\bar{5}$	$\bar{6}$	$\bar{3}$
					$\bar{7}$	$\bar{8}$	4
	3)			$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$
&c				$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{2}$
				$\bar{7}$	$\bar{8}$	$\bar{9}$	$\bar{3}$
	4)		$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{1}$
			$\bar{5}$	$\bar{6}$	$\bar{7}$	$\bar{8}$	$\bar{2}$
	5)	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{1}$
		$\bar{6}$	7	8	9	10	$\bar{2}$
		40	30	20	10	0	

In hac Tabella Diviſores ſunt à ſiniſtrâ intra lineam flexam.

Tum verùs dextram ſequuntur Logarithmorum dividendorum Indices negativi.

His in ſingulis ordinibus collaterales adſtant Quotorum Indîces etiam negativi.

Subtùs autem qui ſcribuntur numeri, 0, 10, 20, 30, 40, Oſtendunt numeros addendos primæ figuræ Logarithmi dividendi, cujus Index negativus invenitur ſupra in eâdem columnâ, juxta Diviſorem. Ut ſi Logarithmus

garithmus $\bar{7}$,41842 poſtuletur dividi per 3: Quæratur $\bar{7}$ juxta 3) dabiturque collateralis $\bar{3}$, pro Indice Quoti: Et numerus 20 ſubtùs; qui additus figuræ dividuæ primæ 4, reddit ipſum 24: in quo Diviſor 3 Octiès coutinetur.

Diviſio.

3) $\bar{7}$,41842	2) $\bar{5}$,61228
Latus $\bar{3}$,80614	Latus, $\bar{3}$,80614

De

De Anatociſmo, ſive Uſura compoſita.

Hoc eſt, Sex Theorematum fundamentalium, quibus Quæſtiones omnes circa Anatociſmum, facili negotio, ſolvi poterunt; inveſtigatio Analytica.

Notandum autem eſt, quod Solidus Anglicus continet 12 Denarios: Et Libra ſive Mina continet 20 Solidos; Denarios verò 240.

1. Ratio fœnoris reducenda primò eſt ad Rationem æqualem, cujus antecedens ſit 100, vel 1. Ut ſi Ratio ſit Denariorum 19,2, vel Solidorum 1,6, pro 1 Libra: Dic, 240. 259,2, vel 20. 21,6::100. 108::1.1,08: nempe α. β. Quare β procreatur ex ſorte α, in uno anno integro.

2. Si vero Solutio ſit per ſemiſſem anni, vel per Quadrantem, hoc eſt per Dies 182,5; vel per Dies 91,25: Pro β, multiplicetur Logarithmus Procreat; annui per $\frac{1}{2}$, vel per $\frac{1}{4}$: Sive & per $\frac{182,5}{365}$ vel per $\frac{91,25}{365}$.
Perperam enim vulgò ſumitur $\frac{1}{2}$ vel $\frac{1}{4}$ annui fœnoris.

3. Quia in progreſſione, numerus Rationum unitate minor eſt, quàm N numerus terminorum, ſive

Solutionum; erit numerus Rationum N−1. Item Logarithmus β ductus in N−1, erit Logarithmus ω ultimi termini. Denique Logarithmus β ductus in N, erit Logarithmus $\beta\omega$, hoc eſt, ipſius β multiplicati in ſe continuè pro numero Solutionum.

4. Quare $\beta\omega$ procreatur ex α ſorte, ſive 1 lb, elocata pro N vicibus. Hinc Duo oriuntur Theoremata.

Theo: I. 1 lb. $\beta\omega$ :: Q lb. Q lb cum lucro in N vicibus.

Theo: II. $\beta\omega$: 1 lb :: Q lb poſt N vices. valor præſens.

5. Deinde quia $\frac{\beta\omega-\alpha q}{\beta-\alpha}$, hoc eſt, $\frac{\beta\omega-1}{\beta-1}=Z$, ſummæ omnium terminorum Progreſſionis (quorum ultimus eſt ω) eſtque idcirco Procreatum ex Penſione 1 lb intermiſſa pro N vicibus: Hinc Duo oriuntur alia Theoremata.

Theo: III. $\beta-1.\ \beta\omega-1$ :: Q lb Penſio intermiſſa pro N vicibus. Penſiones cum fœnore ſolvendæ in fine.

Theo: IV. $\beta\omega-1.\ \beta-1$ :: Q lb futura. Penſio æquivalens ſolvenda in N vicibus.

6. Denique quia $\beta\omega$ procreatur ex 1 lb elocata pro N vicibus: Eſtque $\frac{\beta\omega-1}{\beta-1}$ procreatum ex Penſione 1 lb intermiſſa pro N vicibus; quod in pecuniis numeratis æquivalet pretio Penſionis: Dic, $\beta\omega$. 1 lb :: $\frac{\beta\omega-1}{\beta-1}$. $\frac{\beta\omega-1}{\beta-1 \text{ in } \beta\omega}$: Unde igitur in N vicibus procreabitur $\frac{\beta\omega-1}{\beta-1}$

Pretium

Pretium Penſionis. Hinc etiam oriuntur duo Theoremata.

Theo: V. $\beta-1$ in $\beta\omega.\beta\omega-1$::Q[lb] Penſio pro N vicibus. Pretium ejuſdem in pecuniis numeratis.

Theo: VI. $\beta\omega-1.\beta-1$ in $\beta\omega$:: Q[lb] præſens. Penſio emenda pro N vicibus.

Nota quod Q[lb] ſignificat quantamlibet librarum ſummam.

Exemplum de Penſione durante 10 annos, ſolutione ſemeſtri; in Ratione 1 ad 1,08: Eſtque N 20. Et Logar: 1,08 eſt 0,03342.

0,03342 in $\frac{1}{2}$
0,01671 Log: β=1,0396
20 N
0,33420 Log: $\beta\omega$=2,159
$\bar{2}$,59769 Log: $\beta-1$ =0,0396
$\bar{2}$,93190 Log: $\beta-1$ in $\beta\omega$
0,06408 Log: $\beta\omega-1$=1,159

Eſt igitur,

0,06408
$\bar{2}$,93190
1,13218 Logar: pretii 13,56[lb] pro Penſ: 1[lb].
$\bar{2}$,93190
0,06408
$\bar{2}$,86782 Logar: Penſionis 0,07376[lb] pro Pret: 1[lb].

Solutionum; erit numerus Rationum N−1. Item Logarithmus β ductus in N−1, erit Logarithmus ω ultimi termini. Denique Logarithmus β ductus in N, erit Logarithmus $\beta\omega$, hoc eſt, ipſius β multiplicati in ſe continuè pro numero Solutionum.

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5. Deinde quia $\frac{\beta\omega-\alpha q}{\beta-\alpha}$, hoc eſt, $\frac{\beta\omega-1}{\beta-1}=Z$, ſummæ omnium terminorum Progreſſionis (quorum ultimus eſt ω) eſtque idcirco Procreatum ex Penſione 1 lb intermiſſa pro N vicibus: Hinc Duo oriuntur alia Theoremata.

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Pretium

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Theo: V. $\beta-1$ in $\beta\omega.\beta\omega-1$::Q[lb] Penſio pro N vicibus. Pretium ejuſdem in pecuniis numeratis.

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Nota quod Q[lb] ſignificat quantamlibet librarum ſummam.

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0,03342 in $\frac{1}{2}$
0,01671 Log: β=1,0396
20 N
0,33420 Log: $\beta\omega$=2,159
$\bar{2}$,59769 Log: $\beta-1$ =0,0396
$\bar{2}$,93190 Log: $\beta-1$ in $\beta\omega$
0,06408 Log: $\beta\omega-1$ =1,159

Eſt igitur,

0,06408
$\bar{2}$,93190
1,13218 Logar: pretii 13,56[lb] pro Pens: 1[lb].
2,93190
0,06408
$\bar{2}$,86782 Logar: Penſionis 0,07376[lb] pro Pret: 1[lb].

Logarithmis hiſce inventis adde Logar: Q^{ib}.
Vel valores hoſce inventos multiplica per Q^{ib}.

REGULA FALSÆ POSITIONIS.

Multiplica Poſitiones per alternos errores. Et ſi errores ſint ejuſdem generis, nempe uterque excedens, vel uterque deficiens; Differentiam productorum divide per Differentiam errorum; Si verò diverſi ſint generis; Summam productorum divide per Summam errorum: Et Quotus dabit numerum quæſitum.

Demonſtrationis gratia. Quis numerus eſt, qui ductus in B, producit planum BA, nempe BApl.

Eſto A-C	Eſto A-D.
in B. BA-BC	in B. BA-BD

Errores igitur ſunt

BApl-BA+BC. BApl-BA+BD.

Quia utrobique ſigna ſunt ſimilia; ut quæ æqualia ſunt, expurgentur; opus eſt ut Subductio fiat, mutando omnia ſigna minoris. Nam ſic æqualibus ſe mutuò elidentibus, manebit errorum Differentia, BC-BD.

BC defic:	BD defic:
A-D	A-C
BCA-BCD	BDA-BDC.

Hîc

Hic etiam æqualibus utrinque per Subductionem expunctis; Reductio fit ad BCA-BDA: quæ eſt ipſa errorum differentia ducta in A.

$$\text{Quare } \frac{BCA-BDA}{BC-BD}=A.$$

Iterum Eſto A+C. Eſto A-D
in B. BA+BC in B. BA-BD
Errores igitur ſunt.
BA+BC-BA pl. BA pl-BA+BD.

Quia utrobique ſigna ſunt contraria; æqualia per Additionem, abſque ulla ſignorum mutatione, ſe mutuò elident: Et ſic manebit errorum ſumma, BC+BD.

BC exced:	BD defic:
A-D	A+C
BCA-BCD	BDA+BCD.

Hic etiam æqualibus utrinque per Additionem expunctis; Reductio fit ad BCA+BDA: quæ eſt ipſa errorum ſumma ducta in A.

$$\text{Quare } \frac{BCA+BDA}{BC+BD}=A.$$

Notæ ſeu ſymbola quibus in ſequentibus utor:

Æquale ═. Simile *Sim.*
Majus ⊏. Proxime majus ⊏.
Minus ⊐. Proxime minus ⊐.
Non majus ⊏. Æquale vel minus ⊑.
Non minus ⊐. Æquale vel majus ⊒.
Proportio, ſive ratio æqualis : :
Major ratio ∹. Minor ratio ∻.
Continuè proportionales ∺.
Commenſurabilia ⊓.
Incommenſurabilia ⊓:
Commenſurabilia potentiâ ⊓.
Incommenſurabilia potentiâ ⊓.
Rationale, ῥητὸν, R, vel r.
Irrationale, ἄλογον, $\bar{r}$.
Medium ſive mediale m.

Linea ſecta ſecundum extremam & mediam rationem } $\sigma\tau$ 5

Major ejus portio σ
Minor ejus portio τ.

Z eſt

Z eſt A + E. ʒ eſt a + e.
Z eſt Aq+Eq. ʒ eſt aq + eq. ʒ
X eſt Aq−Eq. ꝭ eſt aq−eq. ꝭ
Æ eſt A E rectang. æ eſt a e rectangulum.
▯ rectangulum. ▫ quadratum.
△ Triang: ℞ latus.
media proportionalis, *m*.

Notæ ſeu ſymbola quibus in ſequentibus utor :

Æquale $=$. Simile *Sim.*
Majus ⊏. Proxime majus ⊏̄.
Minus ⊐. Proxime minus ⊐̄.
Non majus ⊏̸. Æquale vel minus ⊑.
Non minus ⊐̸. Æquale vel majus ⊒.
Proportio, ſive ratio æqualis : :
Major ratio $\div\cdot$. Minor ratio $\cdot\div$.
Continuè proportionales $\div\!:$.
Commenſurabilia ⊓.
Incommenſurabilia ⊔.
Commenſurabilia potentiâ ⊓̣.
Incommenſurabilia potentiâ ⊔̣.
Rationale, ῥητὸν, R, vel ṙ.
Irrationale, ἄλογον, ṙ̸.
Medium ſive mediale ṁ.
Linea ſecta ſecundum extremam & mediam rationem } σ ς
Major ejus portio σ
Minor ejus portio τ.

Z eſt

Z eſt A + E. ζ eſt a + e.
Z eſt Aq+Eq. ζ, eſt aq + eq. ζ,
X eſt Aq-Eq. ꝏ, eſt aq-eq. ꝏ,
Æ eſt A E rectang. æ eſt a e rectangulum.
▯ rectangulum. □ quadratum.
△ Triang: ℒ latus.
media proportionalis, m.

Notæ ſeu ſymbola quibus in ſequentibus utor:

Æquale ═. Simile *Sim.*
Majus ⊏. Proxime majus ⊏.
Minus ⊐. Proxime minus ⊐.
Non majus ⊏. Æquale vel minus ⊏.
Non minus ⊐. Æquale vel majus ⊐.
Proportio, ſive ratio æqualis ::
Major ratio ÷. Minor ratio ÷.
Continuè proportionales ∺.
Commenſurabilia ⊓.
Incommenſurabilia ⊓:
Commenſurabilia potentiâ ⊓.
Incommenſurabilia potentiâ ⊓.
Rationale, ῥητὸν, R, vel ℞.
Irrationale, ἄλογον, ℞.
Medium ſive mediale m.

Linea ſecta ſecundum extremam & mediam rationem } σ 5

Major ejus portio σ
Minor ejus portio τ.

Z eſt

Z eſt A + E.	ʒ eſt a + e.
Z eſt Aq+Eq.	ʒ eſt aq + eq. ʒ
X eſt Aq-Eq.	ꭓ eſt aq-eq. ꭓ

Æ eſt A E rectang. æ eſt a e rectangulum.

▯ rectangulum. ▫ quadratum.

△ Triang: ℒ latus.

media proportionalis, *m*.

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Notæ ſeu ſymbola quibus in ſequentibus utor :

Æquale ═ .   Simile *Sim.*
Majus ⊏ .   Proxime majus ⊏ .
Minus ⊐ .   Proxime minus ⊐ .
Non majus ⊏ .   Æquale vel minus ⊑ .
Non minus ⊐ .   Æquale vel majus ⊒ .
Proportio, ſive ratio æqualis : :
Major ratio ⩪ .   Minor ratio ⩫ .
Continuè proportionales ∺ .
Commenſurabilia ⊓ .
Incommenſurabilia ⊓ .
Commenſurabilia potentiâ ⊓ .
Incommenſurabilia potentiâ ⊓ .
Rationale, ῥητὸν, R, vel ϰ.
Irrationale, ἄλογον, ϰ .
Medium ſive mediale *m* .
Linea ſecta ſecundum extremam & mediam rationem } σ ς
Major ejus portio σ
Minor ejus portio τ.

Z eſt

Z eſt A + E. ʒ eſt a + e.
Z eſt Aq+Eq. ʒ eſt aq + eq. ʒ
X eſt Aq−Eq. Ꝩ eſt aq−eq. Ꝩ
Æ eſt A E rectang. æ eſt a e rectangulum.
▯ rectangulum. ◻ quadratum.
△ Triang: ℞ latus.
media proportionalis, ℳ.

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Notæ ſeu ſymbola quibus in ſequentibus utor :

Æquale $=$. Simile *Sim.*
Majus $\sqsubset$. Proxime majus $\sqsubset\!\!\cdot$.
Minus $\sqsupset$. Proxime minus $\cdot\!\!\sqsupset$.
Non majus $\not\sqsubset$. Æquale vel minus $\sqsubseteq$.
Non minus $\not\sqsupset$. Æquale vel majus $\sqsupseteq$.
Proportio, ſive ratio æqualis : :
Major ratio $\overline{\cdot\cdot}$. Minor ratio $\underline{\cdot\cdot}$.
Continuè proportionales $\div\!\!\div$.
Commenſurabilia ⊓ .
Incommenſurabilia ⊓ :
Commenſurabilia potentiâ ⊓ .
Incommenſurabilia potentiâ ⊓ .
Rationale, ῥητὸν, R, vel $\imath r$.
Irrationale, ἄλογον, $\imath r$.
Medium ſive mediale $m\!r$.

Linea ſecta ſecundum extremam & mediam rationem } σ 5

Major ejus portio σ
Minor ejus portio τ .

Z eſt A + E. ʒ eſt a + e.
Z eſt Aq+Eq. ʒ, eſt aq + eq. ʒ,
X eſt Aq-Eq. ℭ, eſt aq-eq. ℭ,
Æ eſt A E rectang. æ eſt a e rectangulum.
▯ rectangulum. ◻ quadratum.
△ Triang: ℛ latus.
media proportionalis, ṁ.

ELEMENTI DECIMI
EUCLIDIS
Declaratio.

AD def: 1. Eandem menſuram duas magnitudines metiri, tum dicit, quando ipſarum quoti menſuras certas, & veris numeris explicabiles habent. Commenſurabiles igitur magnitudines ſunt, quarum ratio in veris numeris dari poterit: quales ſunt in genere quadratico, radices quadratæ planorum ſimilium: & in genere cubico, radices cubicæ ſolidorum ſimilium. Exempli gratiâ, in planis 18 & 50, nempe 3×6, & 5×10, ſimilibus (eſt enim 3.6::5.10) √q 18, & √q50 ſunt latera commenſurabilia; quia diviſa √q2 maximam utriuſque communem menſuram, dant √q 9 & √q 25, hoc eſt 3 & 5. Sunt igitur √q 18 & √q 50 in ratione 3 ad 5.

Ad def: 2. √q 12 & √q 64 ſunt latera incommenſurabilia: nam quamvis ad minores terminos poterint reduci per √q 4 maximam utriuſque communem

nem menſuram; fientque √q 3 & √q 16 : non tamen dicuntur commenſurabilia; quia non ſunt ut numerus ad numerum. Eſt enim √q 3 numerus non verus, ſed ſurdus.

Ad def: 3. At vero linearum ſive laterum √q 12 & √q 64, quadrata 12 & 64 ſunt commenſurabilia; quia area 1 utrumque metitur : nam area 12, aream 1 continet duodecies; & area 64, ipſam aream 1, continet ſexagies & quater. Quare quadrata illorum laterum ſunt in ratione 12 ad 64. Atque hinc ſequitur quod omne latus ſurdum generis quadratici numero rationali, ſive vero cuicunque potentiâ eſt commenſurabile : modo ſi intelligantur ejuſdem eſſe generis ſive dimenſionis : At ſi unum ex iis intelligatur eſſe latus ſive linea, & alterum planum ſive ſuperficies, non erunt commenſurabilia potentiâ.

Ad def: 4. Sunt igitur lineæ potentiâ incommenſurabiles diverſorum generum : nempe una lateralis, altera quadratica; vel una quadratica, altera quadrato-quadratica. Exempli gratia, laterum √q 3 & √q 2 quadrata ſunt 3 & 2 : & inter ipſa planum medium proportionale √q 6. Quare plana ſive potentiæ 3 & 2 incommenſurabilia ſunt ad planum √q6. Ideoque ipſorum latera √q3 & √q 2 ad √qq6 ſunt incommenſurabilia etiam potentiâ. Atque hujuſmodi media proportionalia tum plana, tum latera, Euclides poſtea Media ſive Medialia nuncupat.

√q3. √q2.
3. √q6. 2.
√q3. √qq6. √q2.

Ad

Ad def. 5. Si linea propoſita vero numero ſit explicabilis; omnes lineæ veris numeris explicabiles, ſunt ipſi commenſurabiles. Si verò linea propoſita ſit latus ſurdum, puta $\sqrt{q}\,3$, linea illi quacunque ratione commenſurabilis invenitur per proportionem. Ut ſi ratio data ſit 2 ad 5: Dic 2. 5 :: $\sqrt{q}\,3 \cdot \sqrt{q}\,\frac{75}{4}$.

Dicitur ῥητὴ, ſive rationalis linea vero numero explicabilis; ratione cujus aliæ lineæ ad ipſam comparatæ, conſiderantur vel commenſurabiles vel incommenſurabiles, idque longitudine vel potentiâ.

Atque his bene perſpectis, reliquæ definitiones nihil habebunt difficultatis.

Sequuntur Lemmata.

1. Rectangulum ſub ɪr & ɪr eſt ɪr. Nam irrationalium aggregatio quantacunque non mutat ſpeciem.

Si linea Z ſecetur inæqualiter in A & E, erit Z − 2AE = Xq. Et Z + 2AE = Zq.

3. Si lineæ Z componatur tum ex A+E, tum ex a+e: erit $Z - \mathfrak{z}_1 = 2æ - 2Æ$.

Item, ſi linea X conſtituatur tum ex A−E, tum ex a−e: erit $Z - \mathfrak{z}_1 = 2Æ - 2æ$.

4. A.E :: Aq.Æ :: Æ.Eq.

5. Si A & E ſint ⊤: erunt 1°, Aq, Eq, Z, X ⊓⊓: ideoque ſimul ɪr vel mr.

Erunt 2°, Aq, Eq, Z, X ⊓⊓ 2Æ. per 4.

Erunt 3°, Z, 2Æ, Zq, Xq ⊓⊓.

Erunt 4°, X, 2Æ, Zq, Xq ⊓⊓. Nam Zq =

Z

Z+2Æ: & 'Xq = Z − 2Æ. & Zq = 4Æ+Xq.

6. Si A & E '⊣, erunt Aq, Eq Zq, Æ, Z, X, Xq '⊓⊓.

Propoſitiones Elementi X[i].

9. Novem priores propoſitiones docent lineas commenſurabiles eſſe, ut numerus ad numerum, atque ideo eorum quadrata eſſe, ut quadratus numerus ad quadratum numerum. In incommenſurabilibus autem contra eſſe. Earum enim quadrata non ſunt ut Q.Q. √q 45 & √q 20 ſunt lineæ commenſurabiles, quia ipſarum quadrata 45 & 20 ſunt ut 9 & 4, numeri quadrati, quorum radices ſunt 3 & 2, ſunt igitur √q 45 & √q20 in ratione 3 ad 2.

Coroll: ad 9. Lineæ ⊓⊓ ſunt etiam ⊣: at non contra. Sed lineæ '⊣ non ſunt id circo '⊣.

10. Si ſit B.C::D.F. ſintque B, C ⊓⊓ vel '⊓⊓: etiam D, F ⊓⊓ vel '⊓⊓ erunt.

12. 14. Si B ⊓⊓ C, & C, D ⊓⊓ vel '⊓⊓, etiam B, D ⊓⊓ vel '⊓⊓ erunt.

13. Si B ⊓⊓ D; & C '⊓⊓ D: erit B '⊓⊓ C.

Coroll: ad 14. Si B '⊓⊓ C; at B ⊓⊓ D, & C ⊓⊓ F: erit D '⊓⊓ F.

16. 17. A, E, Z ſunt ſimul ⊓⊓ vel '⊓⊓.

11. Invenire B, D '⊓⊓ : & B, C '⊣. Sumantur duo aliqui numeri 3 & 2, qui non ſint ut Q.Q. fiatque 3.2::B.F: Item B.D::D.F. Quare B.F::Bq. Dq. At B, F non ſunt Q.Q: ideoque nec Bq. Dq ſunt ut Q.Q. Ergo B, D '⊓⊓ per 9.

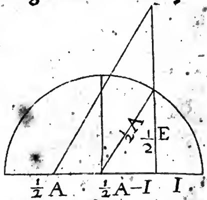
Iterum

Iterum fiat B.C::C.D: ſunt igitur Bq, Cq ⊓ : quare B,C ꝩ. √q 3. √qq 6. √q 2.

Coroll: ad 11. m̃ inter duas ⊓, eſt utrivis ipſarum ꝩ; & ♈, ſi alterutra ex iis ſit ♈.

15. Si ſit A.E:: a.e. & ſit A ⊓ √q:Aq−Eq; ſcil: X: erit etiam a ⊓ √q: aq−eq, ſcil: ♉. Nam Aq.Eq::aq.eq: quare Aq.Aq−Eq::aq.a−eq. Ergo per 20.

18.19. Si ſint duæ lineæ A & E: adplicetur autem ad A rectangulum æquale quadrato ſemiſſis E, deficiens figura quadrata: hoc eſt, dividatur A in duas partes A−I & I, ſic ut ▯^m ſegmentorum æquetur □° ½E; nempe AI−Iq = ¼Eq. Et ſint ſegmenta A−I & I ⊓; erit etiam A ⊓ √q:Aq−Eq. & Converſè: & contra. Nam per 47 e 1, ¼Aq−¼Eq = Q ½ A−I: quare √q:Aq−Eq: eſt A−2I. At per 16 & hypoth: A−2I, & A ſunt ⊓.



22. 23. Ex A, E ♈ ꝩ fit Æ ♈, ſcil: m̃: & √q Æ, eſt ♈ & m̃, (vide annotata ad def. 4). Nam A.E::Aq.Æ quare Æ ⊓ Aq ♈, erit ♈. Eſt etiam Æ m̃. Nam ſi A ſit √q 3, & E √q 2; erit Æ √q 6 planum, cujus radix eſt √qq 6. At vero tam quadrata 3, √q 6, 2; tum ipſorum radices √q 3, √qq 6, √q 2 ſunt ∴ & in neutris medius terminus eſt ejuſdem rationis ſive commenſurationis cum ſuis extremis, ſed utrique incommenſurabilis.

24. Si

24. Si B ſit ᒣ ſaltem ipſi C mr, erit etiam B mr. Nam ad expoſitam R per 23, fiat RD = Cqmr, & RF = Bq. Quare RD ⊔ RF : ideoque FD ır ⊔. Eſt autem per 22, R ⊔ D : idcirco etiam R ⊔ F. Ergo Bq mr : atque ipſa B mr.

20.21.25. Ex A, E ır ⊔, fit Æ ſimiliter ır : & converſè. Et ex A, E mr ⊔, fit Æ mr : & converſè. Nam A.E :: Aq.Æ. At Aq eſt ır vel mr. ergo & Æ ſimiliter ır vel mr, per 24.

26. Ex A, E mr ᒣ, fit Æ ır vel mr. Nam ad expoſitam R, fiat RB = Aq : & RC = Æ : & RD = Eq. Sunt igitur B:D ır ⊔, per 23. Et quia C eſt m∸ inter B & D; erit Cq ır ideoque & ipſa C ır. Si igitur C ır ⊔ R, erit Æ ır. Si vero C ır ⊔ R, erit & Æ mr.

27. Si □□ᵐ B mr conſtet ex □ᵒ Cmr, & □ D: erit etiam □ᵐ D ır. At non converſè. Nam aliter fingatur □ D ır. Ad expoſitam R fiat RA = □ᵐ Cmr ; & RE = □ᵐ D; & RZ = □ B mr. Erit igitur Z ır ⊔ R: & A ır ⊔ R : & E ır ⊔ R. Quare A, E ır ⊔. Eſtque Z ır. At Z ⊔ Zq. Eſt igitur Zq ır, & Z ır : quod oſtenſum eſt falſum. Ergo.

28. Invenire duas A, E mr ᒣ, ita ut Æ ſit ır. Sumantur B, C ır ᒣ : fiatque B.A :: A.C :: C.E. Dico Iᵒ, A, E mr : Nam Aq = BCmr, per 22. Dico IIᵒ, A, E mr ᒣ : Nam B.C :: A.E. Quare per 24. Dico IIIᵒ Æ ır : Nam AE = Cq ır.

29. Invenire duas AE mr ᒣ, ita ut Æ ſit mr. Sumantur B, C, D ır ᒣ : fiatque B.E :: E.D :: A.C. Dico Iᵒ,

I°,A,E mr : Nam Eq=BDmr. D co II°,AEmrꟻ: Nam D. C::E.A. Dico III°, Æmr : Nam AE=BCmr.

Exemplum pro 28. B2. C$\sqrt{}$q3. A$\sqrt{}$qq 12. E$\sqrt{}$qq $\frac{27}{4}$. A E 3.

Exemplum pro 29. B$\sqrt{}$q5. C 2. D$\sqrt{}$q3. E$\sqrt{}$qq15. A$\sqrt{}$qq $\frac{80}{3}$. AE $\sqrt{}$20.

30. Invenire duas A,E irꟻ,ita ut A ⊔ ſit $\sqrt{}$qX. Sumantur duo numeri quadrati aq, eq; ita ut aq−eq non ſit Q. Tum expoſitâ A ir, fiat aq.aq−eq::Aq.Eq. Erit igitur etiam aq.eq::Aq.X, per 19 e 5.

Dico I°,A,E irꟻ : Nam Aq,Eq non ſunt ut Q.Q.

Dico II°, A ꟻ $\sqrt{}$q X : Nam ſunt ut Q.Q.

Exemplum pro 30. Aq & aq ſunt 9. eq & X 4.

31. Invenire duas A, E irꟻ, ita ut A ⊔ $\sqrt{}$qX. Sumantur duo numeri aq, eq, quadrati; ita ut aq+eq non ſit Q. Tum expoſita A ir,fiat aq+eq.aq:: Aq.Eq. Erit igitur aq+eq. eq::Aq.X, per 19 e 5.

Dico I°, A, E irꟻ : Nam Aq, Eq non ſunt ut Q.Q.

Dico II°, A ⊔ $\sqrt{}$qX : Nam Aq, X non ſunt ut Q.Q.

Exemplum pro 31 aq & eq 4. eq & X 1.

32. Invenire duas A, E mrꟻ, ita ut Æ ſit ir; & A ⊔ $\sqrt{}$qX. Summantur per 30, duæ a, e irꟻ, ita ut a ⊔ $\sqrt{}$q: aq−eq. fiatque a.A::A.e::e.E. Dico I°, A,Emrꟻ, per 22 & 24. Nam Aq=aemr: & a.e::A. E,ꟻ. Dico II°, Æ ir : Nam AE=Eq ir. Dico III°, A⊔$\sqrt{}$qX, per 15. Nam a⊔$\sqrt{}$q: aq−eq. Exemplum

plum a 2. e √q3. A √qq 12. E √qq $\frac{27}{4}$.

Quod ſi per 31, Sumerentur a e ɣ⌐ꝗ; ita ut a ⊐ √q: aq−eq: Inventæ fuerint A, E *m*⌐ꝗ, ita ut Æ ſit ɣ; & A ⊐ √qX.

Exemplum a √q5. e 2. A √q 20. E √q $\frac{64}{5}$.

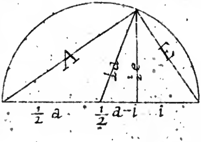
33. Invenire duas A, E *m*⌐ꝗ, ita ut Æ ſit *m*; & A ⊐ √q X. Sumantur per 30, duæ a, e ɣ⌐ꝗ; ita ut a ⊐ ſit aq−eq: & ſumantur i ɣ⌐ꝗ utrique a e: fiatque a.A::A.i::e.E. Dico 1°, A, E *m*⌐ꝗ: Nam Aq = a i *m*: Eſtque a.e::A.E. Dico 11° Æ *m*. Nam AE = i e *m*. Dico 111°, A ⊐ √qX: Nam a ⊐ √q aq−eq: quare per 15.

Exemplum a 2. e √q5. i √q 2. A √qq8. E √qq $\frac{5}{2}$. Quod ſi per 31, ſumerentur a, e ɣ⌐ꝗ, ita ut a ⊐ √q: aq−eq; Inventæ fuerint A, E *m*⌐ꝗ, ita ut Æ ſit *m*: & A ⊐ √qX.

Exemplum, a √q5. e √q2. A √qq 20. E √qq $\frac{16}{5}$.

Præparatio ad propoſitiones 34, 35, 36, demonſtrandas in tribus lemmatibus.

Lemma primum. Si ad a adplicetur rectangulum æquale Q. $\frac{1}{2}$e, deficiens figura quadratâ: diviſâ ſcil: a in a−i & i; ita ut a−i. $\frac{1}{2}$e:: $\frac{1}{2}$ e.i. Erit $\frac{1}{2}$ a−i = √u: $\frac{1}{4}$ aq−$\frac{1}{4}$ eq: ſicut in ſchemate adparet, Atque per hanc interpretationem, a−i = $\frac{1}{2}$a + √u: $\frac{1}{4}$aq−$\frac{1}{4}$eq & i = $\frac{1}{2}$a−√u: $\frac{1}{4}$aq−$\frac{1}{4}$eq. Et quia Aq = Q: a−i: + $\frac{1}{4}$eq. & Eq = iq + $\frac{1}{4}$ eq. Nempe Q. $\frac{1}{2}$ a ± √u: $\frac{1}{4}$aq−$\frac{1}{4}$



eq:

eq: $+\frac{1}{4}$eq. Hac adhibita interpretatione.

Erit A $=\surd$u: $\frac{1}{2}$aq $+\surd$u: $\frac{1}{4}$aqq $-\frac{1}{4}$aqeq.

Et E $=\surd$u: $\frac{1}{2}$aq $-\surd$u: $\frac{1}{4}$aqq $-\frac{1}{4}$aqeq.

Nam in quadratione lineæ $\frac{1}{2}$a $+\surd$u: $\frac{1}{4}$aq $-\frac{1}{4}$eq. Z eſt $\frac{1}{4}$aq $+\frac{1}{4}$aq $-\frac{1}{4}$Eq. Et Æ eſt $\surd$u: $\frac{1}{16}$aqq $-\frac{1}{16}$aqeq: quod duplicatum fiet $\surd$u: $\frac{1}{4}$aqq $-\frac{1}{4}$aqeq. huic ſi adjungatur $+\frac{1}{4}$eq; abolebitur alterum $-\frac{1}{4}$eq.

Lemma ſecundum: a$-$i.i::Aq.Eq, ⊔.

Nam a.A::A.a$-$i } Quare { a.a$-$i::aq.Aq.
Et a.E::E.i } Quare { a.i::aq.Eq.

Lemma tertium: a.A::E.$\frac{1}{2}$e.

34. Invenire duas A,E ⊐, ita ut Z ſit ır, & Æ mr. Sumantur per 31, a,e ır ⊐, ita ut a ⊔ $\surd$q: aq$-$eq: & ex ipſis inveniantur A,E, Sicut in *Lem.pri.*

Dico I° A,E ⊐: Nam per *Lem.ſec:* Aq,Eq ⊔.

Dico II° Z ır: Nam in 31, A,E (quibus hic reſpondent a,e) ſunt ır ⊐.

Dico III°, Æ mr. Nam per *Lem. tert:* AE $=\frac{1}{2}$ ae mr.

35. Invenire duas A,E ⊐, ita ut Z ſit mr & Æ ır. Sumantur per 32, a, e mr ⊐, ita ut æ ſit ır, et a ⊔ $\surd$q: aq$-$eq: Et ex ipſis inveniantur A, E, ſicut in *lem:pri:*

Dico I° A E ⊐, per *lem:ſecun.*

Dico II°, Z mr, per 32.

Dico III°, Æ ır: Nam per *lem:tert:* AE $=\frac{1}{2}$ae ır.

36. Invenire duas A, E ⊐, ita ut Z et Æ ſint mr. Sumantur per 33, a, e mr ⊐, ita ut æ mr, et a ⊔ $\surd$q: aq$-$eq.

Dico

Dico I°, A, E ꝗ, per lem: ſec.

Dico II° Z *mr*, per 33.

Dico III°, Æ *mr*: per lem: tert. conſulatur etiam Schema.

Coroll: ad 36: Hinc inveniri poſſunt duæ lineæ *mr* ⊐, ſcil: √qZ, & √qÆ.

Principium Senariorum per Compoſitionem.

37. Si ſumantur a, e ɤ ꝗ; tota a + e hoc eſt ζ, erit ɤ; vocaturque *Binomium*, ſcil: ♃ *Bin:* I. Nam per lemma 5, ζq ⊐ ζ_1 ɤ.

2 + √q3. Cujus Q: eſt 7 + √q48.

38. Si ſumantur a, e *mr* ꝗ (per 28) ita ut æ ſit ɤ; tota ζ erit ɤ; vocaturque *Bimediale* prius, ſcil: ♃ *Bin:* II. Nam per lemma 5, ζq ⊐ æ ɤ.

√qq 12 + √qq $\frac{27}{4}$. Cujus Q: eſt √q $\frac{147}{4}$ + 6.

39. Si (per 29) ſumantur a, e *mr* ꝗ, ita ut æ ſit *mr*: tota ζ erit ɤ; vocaturque *Bimediale* poſterius, ſcil: ♃ *Bin:* III. Nam ζq, hoc eſt ζ_1 + 2 æ, eſt ɤ. Nam expoſitâ R, fiat RT = ζq; & RP = ζ_1 *mr*, per 16 & 14: Erit RT-RP = 2 æ. Sunt autem per lem: 5, RP & RT-RP *mr* ꝗ: Quare P, T-P ɤ ꝗ ad R. Et per 37, T eſt ɤ. Et per lem: 1, RT hoc eſt ζq ɤ.

√qq $\frac{20}{3}$ + √qq 15. Cujus Q: eſt √q $\frac{245}{3}$ + √q 80.

40. Si (per 34) ſumantur a, e ⊐, ita ut ζ_1 ſit ɤ, & æ *mr*; tota ζ erit ɤ; vocaturque Major, ſcil: ♃ *Bin:* IV. Nam per lem: 6, ζq ⊐ ζ_1 ɤ. √u: $\frac{5}{2}$ + √q $\frac{5}{4}$: pl: √u: $\frac{5}{2}$ − √$\frac{5}{4}$. Q: eſt 5 + √q 20.

41. Si (per 35) ſumantur a, e ꝗ, ita ut ζ_1 ſit *mr*,

[illegible] *Potens rationale & mediale.* [illegible] per lem: 6, [illegible] q [illegible] x [illegible].
[illegible] Q. est √q 20 + 4.

[illegible] & x sint [illegible]; [illegible] *Potens duo medialia.* Scil. [illegible] est [illegible]. Expoſita [illegible] RP = [illegible]; erit RT-RP = [illegible] RT & RT-RP [illegible]: Quare [illegible] Et per 37 T eſt [illegible]. Et [illegible] Ergo [illegible].
[illegible] Q. est √q 20 + √q 8

[illegible] Neque [illegible] ex dictis ſex lineis [illegible] nomina z, e, præterquam [illegible] Nam [illegible] dividatur iterum [illegible] Erit (per lem: 1) Z - [illegible] = 2x - [illegible] I. IV, Z - [illegible] eſt [illegible]; & [illegible] *Bin:* II. V, [illegible] Quare eadem quantitas [illegible] In [illegible] vero *Bin:* III, [illegible] dividi in [illegible] & RP = [illegible], & RT-RP = 2x; [illegible] nomina P, T - P [illegible]. [illegible] diviſi in A, E, alia nomina [illegible] & RS = Z & RT-RS = 2Æ; [illegible] T. dividi iterum in nomina [illegible] & T - P : quod eſt [illegible] *demonſtrationis*. Eſt [illegible]

Defini-

	Definitiones	&	Proprietates
	2℮ *Binom: & Apotom:*		*Binomiorum & Apotom.*
I	ι, e ɼ ⊐⊏ : æ m̃r		A ⊐⊏ √qX : A ⊐⊏ R
II	a, e m̃r ⊐⊏ : æ ɼ		A ⊐⊏ √qX : E ⊐⊏ R
III	a, e m̃r ⊐⊏ : æ m̃r		A ⊐⊏ √qX : AE ⊐⊏ R
IV	a, e ⊐⊏ : ʒ ɼ : æ m̃r		A ⊐⊏ √qX : A ⊐⊏ R
V	a, e ⊐⊏ : ʒ m̃r : æ ɼ		A ⊐⊏ √qX : E ⊐⊏ R
VI	a, e ⊐⊏ : ʒ & æ m̃r ⊐⊏		A ⊐⊏ √qX : AE ⊐⊏ R

(I–III: ⊐⊏ ɼ; IV–VI: AE)

49.50.51.52.53.54. Invenire ſex *Binomia* A+E. Sumatur N [9] & dividatur tum in 5 & [4]; tum in 6 & 3: & exponatur R.

Pro *Bin:* I.IV. Sit A ⊐⊏ R; fiatque [9] ⁵₆ :: Aq.Eq.

Pro *Bin:* II.V. Sit E ⊐⊏ R; fiatque ⁵₆ [9] :: Eq.Aq.

Pro *Bin:* III.VI. Sumatur tertius N (2), qui nec ad 9, nec ad 5, nec ad 6, ut Q.Q. fiatque 2. [9] :: Rq.Aq ɼ. Deinde [9] ⁵₆ :: Aq.Eq ɼ: qui non ſunt ut Q.Q. Quare in omnibus ſex ſunt A.E ɼ ⊐⊏. Item [9] [4/3] :: Aq. X.

55.56.57.58.59.60. Si ſingula ſex *Binomia* A+E ducantur in expoſitum R: √q:AR+ER: conſtituet ordine ſingulas ſpecies 2℮ *Binom.* Nam (conſideratis prius intente proprietatibus cujuſque tum *Binomii*, tum 2℮ *Bin:* in tabella præmiſſa) dividatur A in A−I & I, ita ut AI−Iq=¼Eq. Erit igitur A−I. ½ E :: ½ E.I. fiat etiam aq=AR−IR: & eq=IR.

& æ ir; tota ʒ erit ɣr, vocaturque *Potens rationale & mediale*, ſcil: ♃ *Bin:* V. Nam per lem: 6, ʒq ⊐ æ ir.
√u:√q5 + 1: pl: √u:√q5 − 1. Q: eſt √q20 + 4.

42. Si (per 36) a, e ꟻ, ita ut ʒ & æ ſint mr ꟻ; tota ʒ erit ɣr, vocaturque *Potens duo medialia*. Scil. ♃ *Bin:* VI. Nam ʒq, hoc eſt ʒ + 2æ eſt ɣr. Expoſita enim R, fiant RT = ʒq, & RP = ʒ; erit RT-RP = 2æ. Sunt autem RT & RT-RP mr ꟻ: Quare per 22, P, T-P ir ꟻ ad R. Et per 37 T eſt ɣr. Et per lem: 1, RT hoc eſt ʒq ɣr. Ergo ʒ ɣr.
√u:√q5+√q3: pl: √u:√q5−√q3. Q: eſt √q20+√q8

43.44.45.46.27.48. Neque ulla ex dictis ſex lineis ɣr, ʒ poteſt dividi in ſua nomina a, e, præterquam in uno eodemque puncto. Nam aliter dividatur iterum ʒ in ſua nomina A, E. Erit (per lem: 3) Z − ʒ = 2æ − 2Æ. At (per 37 & 40) in ♃ *Bin:* I, IV, Z − ʒ eſt ir; & 2æ − 2Æ mr, per 27. Et (per 38 & 41) in ♃ *Bin:* II. V, Z − ʒ eſt mr; & 2æ − 2Æ ir. Quare eadem quantitas erit ir & ɣr Quod eſt abſurdum. In ♃ vero *Bin:* III, VI, Quoniam in 39 & 42, ſi ſupponatur ɣr ʒ dividi in a, e; fiatque RT = ʒq, & RP = ʒ, & RT-RP = 2æ; demonſtratum eſt ɣr T dividi in nomina P, T-P ir ꟻ. Item ſi iterum ſupponatur ɣr ʒ dividi in A, E, alia nomina; fiatque RT = ʒq, & RS = Z & RT-RS = 2Æ; ſimiliter demonſtrabitur ɣr T. dividi iterum in nomina S & T S ir ꟻ, diviſa ab iis P & T-P: quod eſt contra priorem partem hujus demonſtrationis. Eſt enim ɣr T *Binomium*.

Defini-

Definitiones		&	Proprietates	
♃ *Binom: & Apotom:*			*Binomiorum & Apotom.*	
I	1, e ϰ⊐ : æ mϰ		A⊏√qX : A⊏R	⊐ϰ
II	a, e mϰ⊐ : æ ϰ		A⊏√qX : E⊏R	
III	a, e mϰ⊐ : æ mϰ		A⊏√qX;AE⊏R	
IV	a, e ⊐ : ʒ ϰ : æ mϰ		A⊏√qX : A⊏R	AE
V	a, e ⊐ : ʒ mϰ : æ ϰ		A⊏√qX : E⊏R	
VI	a, e ⊐ : ʒ & æ mϰ⊏		A⊏√qX:AE⊏R	

49.50.51.52.53.54. Invenire ſex *Binomia* A+E. Sumatur N [9] & dividatur tum in 5 & [4]; tum in 6 & 3 : & exponatur R.

Pro *Bin:* I.IV. Sit A ⊏ R; fiatque [9] $\frac{5}{6}$ :: Aq.Eq.

Pro *Bin:* II.V. Sit E ⊏ R; fiatque $\frac{5}{6}$ [9] :: Eq.Aq.

Pro *Bin:* III.VI. Sumatur tertius N (2), qui nec ad 9, nec ad 5, nec ad 6, ut Q.Q. fiatque 2. [9] :: Rq.Aq ϰ. Deinde [9] $\frac{5}{6}$ :: Aq.Eq ϰ : qui non ſunt ut Q.Q. Quare in omnibus ſex ſunt A.E ϰ ⊐. Item [9] [4] :: Aq. X.
3

55.56.57.58.59.60. Si ſingula ſex *Binomia* A+E ducantur in expoſitum R: √q:AR+ER : conſtituet ordine ſingulas ſpecies ♃ *Binom.* Nam (conſideratis prius intente proprietatibus cujuſque tum *Binomii*, tum ♃ *Bin:* in tabella præmiſſa) dividatur A in A-I & I, ita ut AI-Iq = $\frac{1}{4}$Eq. Erit igitur A-I. $\frac{1}{2}$ E :: $\frac{1}{2}$ E.I. fiat etiam aq = AR-IR: & eq = IR.

& æ ɤ; tota ʒ erit ɤ, vocaturque *Potens rationale & mediale*, ſcil: ♃ *Bin:* V. Nam per lem: 6, ʒq ⊐ æ ɤ.
√u:√q5 + 1: pl: √u:√q5 − 1. Q: eſt √q 20 + 4.

42. Si (per 36) a, e ⊐⊣, ita ut ʒ & æ ſint mɤ⊐⊣; tota ʒ erit ɤ, vocaturque *Potens duo medialia.* Scil. ♃ *Bin:* VI. Nam ʒq, hoc eſt ʒ + 2æ eſt ɤ. Expoſita enim R, fiant RT = ʒq, & RP = ʒ; erit RT-RP = 2æ. Sunt autem RT & RT-RP mɤ⊐⊣ : Quare per 22, P, T-P ɤ⊐⊣ ad R. Et per 37 T eſt ɤ. Et per lem: 1, RT hoc eſt ʒq ɤ. Ergo ʒ ɤ.
√u:√q5 + √q3: pl: √u:√q5 − √q3. Q: eſt √q20 + √q8

43.44.45.46.47.48. Neque ulla ex dictis ſex lineis ɤ, ʒ poteſt dividi in ſua nomina a, e, præterquam in uno eodemque puncto. Nam aliter dividatur iterum ʒ in ſua nomina A, E. Erit (per lem: 3) Z − ʒ = 2æ − 2Æ. At (per 37 & 40) in ♃ *Bin:* I, IV, Z − ʒ eſt ɤ; & 2æ − 2Æ mɤ, per 27. Et (per 38 & 41) in ♃ *Bin:* II. V, Z − ʒ eſt mɤ; & 2æ − 2Æ ɤ. Quare eadem quantitas erit ɤ & ɤ Quod eſt abſurdum. In ♃ vero *Bin:* III, VI, Quoniam in 39 & 42, ſi ſupponatur ɤ ʒ dividi in a, e; fiatque RT = ʒq, & RP = ʒ, & RT-RP = 2æ; demonſtratum eſt ɤT dividi in nomina P, T-P ɤ⊐⊣. Item ſi iterum ſupponatur ɤ ʒ dividi in A, E, alia nomina; fiatque RT = ʒq, & RS = Z & RT-RS = 2Æ; ſimiliter demonſtrabitur ɤT. dividi iterum in nomina S & T S ɤ⊐⊣, diviſa ab iis P & T-P : quod eſt contra priorem partem hujus demonſtrationis. Eſt enim ɤT *Binomium.*

Defini-

	Definitiones	&	Proprietates	
	2ℓ *Binom: & Apotom:*		*Binomiorum & Apotom.*	
I	ı, e ır ɥ : æ mr		A ⊓ √qX : A ⊓ R	⊓
II	a, e mr ɥ : æ ır		A ⊓ √qX : E ⊓ R	
III	a, e mr ɥ : æ mr		A ⊓ √qX : AE ⊓ R	
IV	a, e ɥ : ʒ ır : æ mr		A ⊓ √qX : A ⊓ R	AE
V	a, e ɥ : ʒ mr : æ ır		A ⊓ √qX : E ⊓ R	
VI	a, e ɥ : ʒ & æ mr ⊓		A ⊓ √qX : AE ⊓ R	

49.50.51.52.53.54. Invenire ſex *Binomia* A+E. Sumatur N [9] & dividatur tum in 5 & [4]; tum in 6 & 3 : & exponatur R.

Pro *Bin:* I.IV. Sit A ⊓ R; fiatque [9] $\frac{5}{6}$ :: Aq.Eq.

Pro *Bin:* II.V. Sit E ⊓ R; fiatque $\frac{5}{6}$ [9] :: Eq.Aq.

Pro *Bin:* III.VI. Sumatur tertius N (2), qui nec ad 9, nec ad 5, nec ad 6, ut Q.Q. fiatque 2. [9] :: Rq.Aq ır. Deinde [9] $\frac{5}{6}$ :: Aq.Eq ır : qui non ſunt ut Q.Q. Quare in omnibus ſex ſunt A.E ır ɥ. Item [9] [$\frac{4}{3}$] :: Aq. X.

55.56.57.58.59.60. Si ſingula ſex *Binomia* A+E ducantur in expoſitum R: √q:AR+ER : conſtituet ordine ſingulas ſpecies 2ℓ *Binom.* Nam (conſideratis prius intente proprietatibus cujuſque tum *Binomii*, tum 2ℓ *Bin:* in tabella præmiſſa) dividatur A in A-I & I, ita ut AI-Iq = $\frac{1}{4}$Eq. Erit igitur A-I. $\frac{1}{2}$ E :: $\frac{1}{2}$E.I. fiat etiam aq = AR-IR: & eq = IR.

& æ ɼ; tota ζ erit ɼ, vocaturque *Potens rationale & mediale*, ſcil: ℞ *Bin:* V. Nam per lem: 6, ζq ⊐ æ ɼ.
$\surd$u: $\surd$q5 + 1: pl: $\surd$u: $\surd$q5 − 1. Q: eſt $\surd$q 20 + 4.

42. Si (per 36) a, e ⊣, ita ut ζ & æ ſint mr ⊣; tota ζ erit ɼ, vocaturque *Potens duo medialia.* Scil. ℞ *Bin:* VI. Nam ζq, hoc eſt ζ + 2æ eſt ɼ. Expoſita enim R, fiant RT = ζq, & RP = ζ; erit RT−RP = 2æ. Sunt autem RT & RT−RP mr ⊣: Quare per 22, P, T−P ɼ ⊣ ad R. Et per 37 T eſt ɼ. Et per lem: 1, RT hoc eſt ζq ɼ. Ergo ζ ɼ.
$\surd$u: $\surd$q5 + $\surd$q3: pl: $\surd$u: $\surd$q5 − $\surd$q3. Q: eſt $\surd$q20 + $\surd$q8

43. 44. 45. 46. 47. 48. Neque ulla ex dictis ſex lineis ɼ, ζ poteſt dividi in ſua nomina a, e, præterquam in uno eodemque puncto. Nam aliter dividatur iterum ζ in ſua nomina A, E. Erit (per lem: 3) Z − ζ = 2æ − 2Æ. At (per 37 & 40) in ℞ *Bin:* I, IV, Z − ζ eſt ɼ; & 2æ − 2Æ mr, per 27. Et (per 38 & 41) in ℞ *Bin:* II. V, Z − ζ eſt mr; & 2æ − 2Æ ɼ. Quare eadem quantitas erit ɼ & ɼ Quod eſt abſurdum. In ℞ vero *Bin:* III, VI, Quoniam in 39 & 42, ſi ſupponatur ɼζ dividi in a, e; fiatque RT = ζq, & RP = ζ, & RT−RP = 2æ; demonſtratum eſt ɼT dividi in nomina P, T−P ɼ ⊣. Item ſi iterum ſupponatur ɼζ dividi in A, E, alia nomina; fiatque RT = ζq, & RS = Z & RT−RS = 2Æ; ſimiliter demonſtrabitur ɼT. dividi iterum in nomina S & T S ɼ ⊣, diviſa ab iis P & T−P: quod eſt contra priorem partem hujus demonſtrationis. Eſt enim ɼT *Binominum.*

Defini-

	Definitiones	&	Proprietates	
	♃ *Binom:* & *Apotom:*		*Binomiorum* & *Apotom.*	
I	ı, e ʀ ꟻ : æ mr		A ⊐ √qX : A ⊐ R	ʀ ꟻ
II	a, e mr ꟻ : æ ʀ		A ⊐ √qX : E ⊐ R	
III	a, e mr ꟻ : æ mr		A ⊐ √qX ; AE ⊐ R	
IV	a, e ꟻ : ʒ ʀ : æ mr		A ⊐ √qX : A ⊐ R	AE
V	a, e ꟻ : ʒ mr : æ ʀ		A ⊐ √qX : E ⊐ R	
VI	a, e ꟻ : ʒ & æ mr ⊐		A ⊐ √qX : AE ⊐ R	

49.50.51.52.53.54. Invenire ſex *Binomia* A+E. Sumatur N [9] & dividatur tum in 5 & [4]; tum in 6 & 3 : & exponatur R.

Pro *Bin:* I.IV. Sit A ⊐ R; fiatque [9] $\frac{5}{6}$ :: Aq.Eq.

Pro *Bin:* II.V. Sit E ⊐ R; fiatque $\frac{5}{6}$ [9] :: Eq.Aq.

Pro *Bin:* III.VI. Sumatur tertius N (2), qui nec ad 9, nec ad 5, nec ad 6, ut Q.Q. fiatque 2. [9] :: Rq.Aq ʀ. Deinde [9] $\frac{5}{6}$ :: Aq.Eq ʀ : qui non ſunt ut Q.Q. Quare in omnibus ſex ſunt A.E ʀ ꟻ. Item [9] [4]/3 :: Aq. X.

55.56.57.58.59.60. Si ſingula ſex *Binomia* A+E ducantur in expoſitum R: √q:AR+ER : conſtituet ordine ſingulas ſpecies ♃ *Binom.* Nam (conſideratis prius intente proprietatibus cujuſque tum *Binomii*, tum ♃ *Bin:* in tabella præmiſſa) dividatur A in A-I & I, ita ut AI-Iq = ¼ Eq. Erit igitur A-I. ½ E :: ½ E.I. fiat etiam aq = AR-IR: & eq = IR.

& æ ɼ; tota ʒ erit ɿ, vocaturque *Potens rationale & mediale*, ſcil: ♃ *Bin:* V. Nam per lem: 6, ʒq ⊐ æ ɼ.

√u:√q5 + 1: pl: √u:√q5 − 1. Q: eſt √q 20 + 4.

42. Si (per 36) a, e ⊣, ita ut ʒ & æ ſint mɼ ⊣; tota ʒ erit ɿ, vocaturque *Potens duo medialia.* Scil. ♃ *Bin:* VI. Nam ʒq, hoc eſt ʒ + 2æ eſt ɿ. Expoſita enim R, fiant RT = ʒq, & RP = ʒ; erit RT − RP = 2æ. Sunt autem RT & RT − RP mɼ ⊣: Quare per 22, P, T − P ɼ ⊣ ad R. Et per 37 T eſt ɿ. Et per lem: 1, RT hoc eſt ʒq ɿ. Ergo ʒ ɿ.

√u:√q5 + √q3: pl: √u:√q5 − √q3. Q: eſt √q20 + √q8

43. 44. 45. 46. 27. 48. Neque ulla ex dictis ſex lineis ɿ, ʒ poteſt dividi in ſua nomina a, e, præterquam in uno eodemque puncto. Nam aliter dividatur iterum ʒ in ſua nomina A, E. Erit (per lem: 3) Z − ʒ = 2æ − 2Æ. At (per 37 & 40) in ♃ *Bin:* I, IV, Z − ʒ eſt ɼ; & 2æ − 2Æ mɼ, per 27. Et (per 38 & 41) in ♃ *Bin:* II. V, Z − ʒ eſt mɼ; & 2æ − 2Æ ɼ. Quare eadem quantitas erit ɼ & ɿ Quod eſt abſurdum. In ♃ vero *Bin:* III, VI, Quoniam in 39 & 42, ſi ſupponatur ɿ ʒ dividi in a, e; fiatque RT = ʒq, & RP = ʒ, & RT − RP = 2æ; demonſtratum eſt ɿ T dividi in nomina P, T − P ɼ ⊣. Item ſi iterum ſupponatur ɿ ʒ dividi in A, E, alia nomina; fiatque RT = ʒq, & RS = Z & RT − RS = 2Æ; ſimiliter demonſtrabitur ɿ T. dividi iterum in nomina S & T S ɼ ⊣, diviſa ab iis P & T − P: quod eſt contra priorem partem hujus demonſtrationis. Eſt enim ɿ T *Binomium.*

Defini-

Definitiones		&	Proprietates	
♃ *Binom: & Apotom:*			*Binomiorum & Apotom.*	
I	ı, e ᴚ ㄐ : æ m̃		A ⊐ √qX : A ⊐ R	AE ᴚ ㄐ
II	a, e m̃ ㄐ : æ ᴚ		A ⊐ √qX : E ⊐ R	
III	a, e m̃ ㄐ : æ m̃		A ⊐ √qX : AE ⊐ R	
IV	a, e ㄐ : ʒ ᴚ : æ m̃		A ⊐ √qX : A ⊐ R	
V	a, e ㄐ : ʒ m̃ : æ ᴚ		A ⊐ √qX : E ⊐ R	
VI	a, e ㄐ : ʒ & æ m̃ ⊐		A ⊐ √qX : AE ⊐ R	

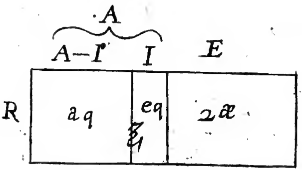
49.50.51.52.53.54. Invenire ſex *Binomia* A+E. Sumatur N [9] & dividatur tum in 5 & [4]; tum in 6 & 3: & exponatur R.

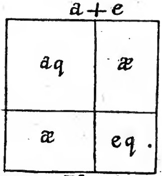
Pro *Bin:* I.IV. Sit A ⊐ R; fiatque [9] $\frac{5}{6}$ :: Aq.Eq.

Pro *Bin:* II.V. Sit E ⊐ R; fiatque $\frac{5}{6}$ [9] :: Eq.Aq.

Pro *Bin:* III.VI. Sumatur tertius N (2), qui nec ad 9, nec ad 5, nec ad 6, ut Q.Q. fiatque 2. [9] :: Rq.Aq ᴚ. Deinde [9] $\frac{5}{6}$ :: Aq.Eq ᴚ: qui non ſunt ut Q.Q. Quare in omnibus ſex ſunt A.E ᴚ ㄐ. Item [9] $\frac{[4]}{3}$ :: Aq. X.

55.56.57.58.59.60. Si ſingula ſex *Binomia* A+E ducantur in expoſitum R: √q:AR+ER: conſtituet ordine ſingulas ſpecies ♃ *Binom.* Nam (conſideratis prius intente proprietatibus cujuſque tum *Binomii*, tum ♃ *Bin:* in tabella præmiſſa) dividatur A in A-I & I, ita ut AI-Iq = ¼ Eq. Erit igitur A-I. ½ E :: ½ E.I. fiat etiam aq = AR-IR: & eq = IR.





Probatur 1°, a + e eſſe √q: AR+ER. Eſt enim AR−IR. ½ ER :: ½ ER . IR: Item aq.æ::æ.eq. Quare ½ ER = æ. Ergo Q: a + e : = AR + ER.

Probatur 11°, In tribus prioribus *Binom:* a, e eſſe ℸ. Nam quia (per 18) AR−IR ⊐ IR, erit AR−IR ⊐ AR: at AR ℸ ER: ergo AR−IR ℸ ER: hoc eſt aq ℸ æ: Eſt autem aq.æ::a.e.

In tribus poſterioribus *Binom:* a, e eſſe ℸ. Nam (per 19) AR−IR, IR, hoc eſt aq, eq ⊐.

Probatur 111°, In *Bin:* 1, a, e eſſe ir. Nam AR−IR, IR, hoc eſt aq, eq ⊐ ſunt AR ir.

In *Bin:* 11, a, e eſſe mr: Nam quia A−I, I ⊐ A ℸ R; Erit AR−IR, IR, hoc eſt aq, eq mr: at a, e ⊐. Item æ eſſe ir: Nam 2æ = ER ir.

In *Bin:* 111, a, e eſſe mr, ut ante. Item æ eſſe ir: Nam ER hoc eſt 2 æ mr, quia E ir ⊐ R.

In *Bin:* IV. aq + eq, hoc eſt AR, eſſe ir. Nam A ir ⊐ R. Item 2 æ, hoc eſt ER, eſſe mr: ut ante.

In *Bin:* V, aq+eq, hoc eſt AR, eſſe mr. Nam A ir ℸ. Item 2 æ, hoc eſt ER, eſſe ir. Nam E ir ⊐ R.

In

In *Bin:* VI, aq+eq, hoc eſt AR; Item 2 æ, hoc eſt ER, eſſe m͞r. Nam A, E ɤ ⊐R.

Atque in omnibus his tribus poſterioribus liquet a & e eſſe ⊐̄, quia aq, eq ⊐.

Conſect: Latus quadratum ſingulorum *Binomiorum* A+E conſtituet ordine ſingulas ſpecies ♃ *Bin:* a+e. Nam poſita R eſſe 1, nihil per multiplicationem immutabitur. Unde majus quadratum erit A−I cujus latus eſt a: & minus quadratum I, cujus latus eſt e. Oſtenſum autem eſt ad prop: 34, in lem: pri: A−I eſſe $\frac{1}{2}$ A + √u: $\frac{1}{4}$ Aq−$\frac{1}{4}$ Eq. Et I eſſe $\frac{1}{2}$ A−√u: $\frac{1}{4}$ Aq−$\frac{1}{4}$ Aq−$\frac{1}{4}$ Eq. Atque hinc patet Analyſis *Binomii;* cujus hæc eſt regula.

Si è quadrato ſemiſſis nominis majoris tollatur quadratum ſemiſſis nominis minoris; & latus quadratum exceſſus ſemiſſi nominis majoris addatur, dabit quadratum majus: ſin detrahatur, minus.

Si igitur ſemis nominis majoris, & latus quadratum exceſſus, ſint commenſurabiles, ♃ *Bin:* erit bimembre. Si incommenſurabiles, quadrimembre.

61.62.63.64.65.66. Si quadratum ex ɤ a + e, ♃ *Bin:* aliquo, ad expoſitum R adplicetur; latitudinem faciet A + E, idem *Binomium.* Nam (ſicut in Schemate ad 55) fiat AR+ER=Q: a + e: Et AR−IR=aq. Et IR=eq: ideoque ER=2æ. Probatur 1°.

In tribus prioribus *Binomiis*, A eſſe ⊐ √qX: Nam a, e ⊐̄; quare AR−IR, IR ⊐. Ergo per 18.

In tribus poſterioribus *Binomiis*, A eſſe ⊐ √qX:

 Nam

Nam a, e ſunt '⊐ : quare AR - IR, IR '⊐. Ergo.

Probatur II°, A, E eſſe ῥ Ч, &c. Nam in *Bin:* I A eſt ῥ '⊐ R; & E ῥ '⊐ R : eſt enim AR, hoc eſt aq+eq ῥ. & ER, hoc eſt 2 æ '⊐ aq+eq, per lem 5.

In *Bin:* II, E eſt ῥ '⊐ R: & A ῥ '⊐ R : Eſt enim ER, hoc eſt 2 æ ῥ. Et AR, hoc eſt aq+eq, '⊐ æ, per lem: 5.

In *Bin:* III, A & E ſunt ῥ '⊐ R: Eſt enim AR, hoc eſt ζ_1 : & ER, hoc 2 æ μ.

In *Bin:* IV, A eſt ῥ '⊐ R : & E ῥ '⊐ R: eſt etiam AR, hoc eſt ζ_1, ῥ; & ER, hoc eſt, 2 æ, μ. Et

In *Bin:* V. VI, ſimiliter ex proprietatibus eorum poterit argui.

67. Si *Binomio* alicui A+E ⊐ ſit B+C; Erit etiam *Binomium* ordine idem. Nam fiat A+E. B+C :: A.B::E.C, ⊐, per 14. & 16. Item per 15, Si A, √q: Aq Eq ⊐ ſit vel '⊐ : Erit etiam B, √q: Bq−Cq : ⊐ vel '⊐.

68. Si in 2℞ *Bin:* II. III, a, e ⊐ b c : Erit *Bimediale* ordine idem. Nam fiat a+e. b+c:: a.b::e.c, ⊐. Sunt autem a, e μ Ч : Ergo b, c, μ Ч per 24. Item a.c::aq æ: Et b.c::bq.bc: quare aq.bq::æ.bc, ⊐. Ergo ſi æ ῥ ſit vel μ; Etiam bc ῥ vel μ erit.

69. 70. 71. Si in tribus poſterioribus 2℞ *Bin:* a, e Ч b, c : Erit 2℞ *Bin:* ordine idem. Nam fiat a+e.b+c::a. b::e.c, Ч ſaltem. Sunt autem a, e 'Ч. Ergo b, c 'Ч. Item quia aq. bq::eq. cq:: aq+eq. bq+cq, Ч ſaltem : Si aq+eq ῥ vel μ; etiam bq+cq. erit ῥ vel μ. Denique quia aq.æ::a.e::b.c::bq.bc; erit

erit aq.bq::æ.bc, ꟻ ſaltem. Si æ ϰ ſit vel ɱ; etiam b c ϰ vel ɱ erit.

72. 73. Si duo ſpacia ζ & 2 æ componantur, quorum unum eſt ϰ, & alterum mediale; ſitque ϰ majus; recta totum ſpacium potens erit ♃ *Bin:*I. vel IV. Sin ɱ majus; recta totum ſpacium potens erit ♃ *Bin:* II, vel V. Si vero duo ſpacia ɱ ⊐ componantur: recta totum ſpacium potens erit ♃ *Bin:* III, vel VI. Nam ſi ad expoſitam R adplicetur AR+ER = ζ + 2 æ, conjunctim & ſeorſim, nempe AR = ζ : & ER = 2 æ; ſive unum ex ipſiis ſit ϰ, & alterum ɱ: ſive utrumque ɱ ꟻ. Clarum erit AR, ER eſſe ⊐; ideoque A, E ϰ ꟻ. Quare ſi A ⊐ √qX, erit A+E unum ex tribus prioribus *Binomiis.* Si verò A ⊐ √q X, erit A+E unum ex tribus poſterioribus *Binomiis.* Quodcunque autem ex ipſiis ſex fuerit; latus illius (quod etiam eſt √u: ζ + 2æ) erit ♃ *Bin:* ordine idem, per 55.56.57.58.59.60.

Principium Senariorum per detractionem.

74.75.76.77.78.79. Si ab *à* majore nomine cujuſvis ♃ *Bin:* auferatur e nomen minus, Reliquum a−e erit ϰ, ♃ *Apotome* ejuſdem ordinis: vocaturque vel *Apotome*, vel *Reſiduum mediale primum, vel Reſiduum mediale ſecundum, vel Minor, vel Cum rationali totum mediale faciens, vel Cum mediali totum mediale faciens.*

Nam Idem probari poteſt de ♉q, quod de ζq probatum

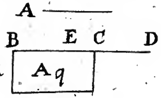
batum fuit, in 37.38.40.41. Sed pro ♃ *Apot:* III. vel VI, ad expoſitam R, fiant RP = ♉q : & RT = ℨ : Et RT-RP = 2æ. Et reliqua fiant ſicut in 39. & 42. Nam ℨ-2æ = ♉q.

80.81.82.83.84.85. Lineis hiſce ſex ♈ a-e, ♃ *Apot:* una tantum congruit recta linea e, pro minore nomine. Nam aliter conſtituatur linea ♉, nempe a-e, etiam ex A-E. Per lem: 3, Z-ℨ = 2Æ-2æ: At in ♃ *Apot:* I. IV, Z-ℨ eſt ♈, & 2Æ-2æ eſt ♍. Et in ♃ *Apot:* II. V, Z-ℨ eſt ♍ : & 2Æ-2æ eſt ♈ (per 37.38 40.41) : quare eadem quantitas eſt ♈ & ♈ : quod eſt abſurdum : In ♃ vero *Apot:* III VI. Quoniam (ſicut eſt in 45 & 48) Si ſupponatur ♈ & ♉ conſtitui ex a, e ; fiatque RP = ♉q; RT = ℨ : & RT-RP = 2æ : demonſtratum eſt ♈ P conſtitui ex nominibus T, T-P, ♈♑. Item ſi iterum ſupponatur ♈ ♉ conſtitui ex A, E aliis nominibus ; fiatque RP = ♉q : RC = ℨ : & RC-RP = 2æ. Similiter demonſtrabitur ♈ P conſtitui ex nominibus C, C-P (diverſis a T & T-P) ♈♑. Quod eſt contra priorem partem hujus demonſtrationis. Eſt enim ♈ P *Apotome.*

86. 87. 88. 89. 90. 91	demonſtrantur verbatim de ♌ ſicut de ℨ.	49.50.51.52.53.54
92. 93. 94. 95. 96. 97		55.56.57.58.59.60
98.99.100.101.102.103		61.62.63.64.65.66
104		67
105. 106. 107.108		68.69.70.71
109. 110. 111		72. 73.

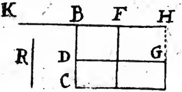
112. Eadem linea ♈ non eſt *Apotome*, & *Binomium.*

mium. Nam eſto A *Apotome*, puta 2ε *Apot:* I: Expoſitâ R, fiat R×BC=Aq. quare per 98 & 61, BC erit *Apotome* 1; ei congruat CD. Sunt igitur BD, CD ɼ┬; & BD ⊓R. Rurſus ponatur A *Binomium*, puta 2ε *Bin:* 1, fiatque R×BC=Aq: Erit per 61 BC *Bin:* 1: cujus nomina ſint BE,CE, ɼ┬; & BE⊓R. Sunt igitur & per 16, BD, BE, ED ɼ⊓: ideoque ED,CD ɼ┬: quare CE *Apot:* ɼ. At CE fuit & ɼ. Quod eſt abſurdum,



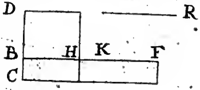
113.114. Rq adplicatum ad *Binomium*, latitudinem facit *Apotomen.* Sed adplicatum ad *Apotomen*, latitudinem facit *Binomium.* Utrobique autem nomina ſunt ⊓ & proportionalia, & utriuſque ordo idem. Nam in utroque Schemate, fiat BC×BF=Rq =DC×BH ɼ⊓. Eſt igitur BC.DC::BH.BF: Et(BC-DC)BD.DC::(BH-BF)FH.BF. His ſic ordinatis.

Pro 113, Eſto *Binomium* aliquod BC, ſcil: BD+DC: fiatque FH--BF.BF::BF.BK. Eſt igitur (BF+BK) FK.BK :: FH.BF :: BD.DC, ɼ┬. Quare FK, BK ┬. Item (FK+FH) HK. FK :: (BK+BF) FK.BK, ┬. Et HK. BK :: HKq. FKq :: FKq :: FKq. BKq ⊓. Unde & per 16, HK,BK,BH ⊓: At BH ɼ: quare HK,BK ɼ⊓: Et FK, BK ɼ┬. Ergo per def: FK-BK, ſcil: BF eſt *Apotome.*



Pro

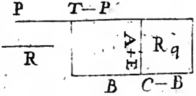
Pro 114. Eſto *Apotome* aliqua BC, ſcil: DC–BD: fiatque FH. BF::HK.FK:: (FH-AK.BF-BK) FK. BK::FH.BF::BD.DC ɼ⊓̲. Quare HK.FK::FK. BK⊓̲: Et HKq, FKq⊓. Unde per 16, HK, BK, BH⊓. At BH ɼ: Itaque BK ɼ, & FK, BK ɼ⊓̲. Ergo per def: BK+FK, ſcil: BF. eſt *Binomium.*



Secundò DC; BK⊓: Et BD, FK⊓. Nam BK ⊓BH⊓DC. Et DC. BK::BD.FK. Ergo Tertio proportionalia.

Quarto ſunt in eodem ordine; per 15 & 14.

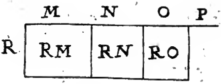
115. Si *Apotomes* T-P, & *Binomii* A+E nomina ſint ⊓ & proportionalia: Nempe T.A⊓::P.E⊓: Dico ▭ T-P in A+E eſſe ɼ. Nam expoſita R, fiat A+E in C-B=Rq. Eſt igitur C-B *Apotome*, per 113: Et A. C⊓ :: E. B⊓: Quare C.T⊓ :: C-B. T-P⊓ :: A+E in C-B ɼ. A+E in T-P etiam ɼ. Et √q: A+E in T-P: ɼ.



116. A Mediali M fieri poterunt innumeræ lineæ ɼ, quæ nec Mediæ ſunt, nec ullæ ex bis ſenis antedictis. Nam expoſita R, fiat MR; & ſit N=√q MR. Dico N eſſe ɼ, per lem: 1: at nec mediale; per 23: nec ullam ex bis ſenis, per 61, 62, 63, 64, 65, 66, & 98, 99, 100, 101, 102, 103.

Deinde

Deinde fiat RN.
Et ſit O $=\sqrt{q}$ RN:
Dico Oꝟ nec Mediale eſſe, nec ullam ex bis ſenis illis.



Tertiò fiat OR, & ſit P $=\sqrt{q}$ OR : Dico Pꝟ eſſe nec Mediale, nec ullam &c. Et ſic infinitum. Neque etiam ꝟ N, O ſunt eædem. Nam N $=\sqrt{q}$ MR. & O $=\sqrt{q}$ NR : &c.

117. Diameter quadrati eſt lateri incommenſurabilis: Nam alias ſi ſit ⊤⊥; eſto Diameter ad Latus, ut numerus D ad numerum L; ſintque minimi termini in eadem ratione : Et ipſorum quadrata ſunt verè numeri quadrati. At verò L non poteſt eſſe 1; quia quadratum diametri ad quadratum lateris eſt ut 2 ad 1 : ideoque 2 eſſet numerus verè quadratus. Nec poteſt L eſſe numerus aliquis multitudinis : quia cùm ſit Dq.L::2.1; & Lq metiatur Dq; etiam L metietur D : ideoque D & L non erunt rationis ſuæ termini minimi: Eſt enim numerus multitudinis maxima utriuſque communis menſura.

Finis Elementi decimi Euclidis.

De

De ſolidis regularibus.

1. **F**Igura quævis polygona rectilinea dividitur in triangula duobus pauciora, quam ſunt numeri laterum. Nempe quadrangulum dividitur in duo triangula : quinquangulum in tria; &c.

2. Quare ſi è numero laterum tollatur 2, & reliquus duplicetur : vel ſi è numero laterum duplicato tollatur 4 : habebis ſummam angulorum rectorum in rectilinea quavis figura interius comprehenſorum. Sic triangulum intra ſe continet duos rectos : quadrangulum quatuor : quinquangulum ſex : &c.

3. Figuræ autem cujuſvis rectilineæ anguli exteriores omnes æquantur quatuor rectis.

4. Quare ſi quatuor anguli recti dividantur per numerum laterum, ſive angulorum : quotus erit quantitas unius anguli exterioris, in figura rectilinea ordinata. Sic angulus exterior in trigono ordinato eſt $\frac{4}{3}$ recti, ſive grad: $\frac{360}{3}$: in tetragono ordinato $\frac{4}{4}$ recti, ſive gradus $\frac{360}{4}$: in pentagono ordinato $\frac{4}{5}$ recti, ſive gradus $\frac{360}{5}$, &c.

5. Si quantitas anguli exterioris tollatur ex duobus rectis : vel ſi ſumma angulorum rectorum interiorum dividatur in numerum laterum : habebis quantitatem unius anguli interioris, in figura rectilinea ordinata. Sequitur pars prior ex 4: poſterior ex 2. Exempli gratia, In octogono ordinato, anguli unius exterioris quantitas eſt $2-\frac{4}{8}$ vel Gra: $180-\frac{360}{8}$. Item 8) 12($1\frac{1}{2}$ recti: vel Gra: 8) 12×90(135.

6. Tres

6. Tres anguli plani recti, vel gradus 270, includunt angulum rectum ſolidum.

7. Quare ſi omnes anguli plani includentes angulum ſolidum, addantur; & aggregatum per 3 angulos rectos, ſive gradus 270, dividatur: habebis quantitatem totius anguli ſolidi. Exempli gratia, Anguli ſolidi Icoſaëdri, quem quinque anguli trigoni ordinati includunt (nempe $5\times\frac{2}{3}$ recti; vel Gra: 5×60) quantitas erit $\frac{10}{9}$ recti; vel Gra: $\frac{5\times60}{270}$.

Rurſus anguli ſolidi Dodacaëdri, quem tres anguli pentagoni ordinata includunt (nempe $3\times\frac{6}{5}$ recti; vel Gra: 3×108) quantitas eſt $\frac{6}{5}$ recti; vel Gra: $\frac{3\times108}{270}$.

Conſect. Atque hoc modo invenientur menſuræ omnium quinque corporum regularium: Nempe Tetraëdri (4), $\frac{2}{3}$ recti, vel $\frac{180}{270}$. Hexaëdri (6), 1 rectum, vel $\frac{270}{270}$. Octaëdri (8), $\frac{8}{9}$ recti vel $\frac{240}{270}$. Icoſaëdri (20), $\frac{10}{9}$ recti, vel $\frac{300}{270}$. Dodecaëdri (12), $\frac{6}{5}$ recti, vel $\frac{324}{270}$. Atque hi ſunt ipſorum characteres.

8. Octo anguli recti ſolidi complent locum ſolidum. Quare ſi angulus ſolidus ſit aliquota pars octo rectorum, vel Gra: $\frac{2160}{270}$: angulus ille ſolidus toties ſumptus complebit locum ſolidum. Nempe anguli Tetraëdri 12: Hexaëdri 8: Octaëdri 9. Nam $\frac{2}{3}$)8(12. Et $\frac{8}{9}$)8(9.

9. Numerus angulorum planorum in ſolido quovis regulari, invenitur multiplicando numerum angulorum planorum unius baſis in numerum baſium. Nempe anguli plani ſunt, in (4), 3×4: in (6), 4×6: in

in (8), 3×8 : in (20), 3×20 : in (12), 5×12.

10. Numerus angulorum ſolidorum in ſolido quovis regulari, invenitur dividendo numerum angulorum planorum in ſolido illo, per numerum angulorum planorum circa unum angulum ſolidum. Nempe anguli ſolidi ſunt, in (4), $\frac{3 \times 4}{3}$: in (6), $\frac{4 \times 6}{3}$, in (8) $\frac{3 \times 8}{4}$: in (20), $\frac{3 \times 20}{5}$: in (12), $\frac{5 \times 12}{3}$.

11. Numerus linearum lateralium in ſolido quovis regulari, eſt ſemis numeri angulorum planorum in illo ſolido. Atque hic etiam eſt numerus rectangulorum, ſub latere, & lineâ perpendiculari è centro baſis in latus. Nam unaquæque linea lateralis duobus inſervit angulis.

12. Quare rectangulum ſub latere, & linea perpendiculari è centro baſis in latus, eſt ſuperficiei totius, in (4), $\frac{1}{6}$: in (6) & (8), $\frac{1}{12}$: in (20) & (12), $\frac{1}{30}$. Eſt 6 & 7 e 14.

13. Solidum quodque regulare æquale eſt ſuperficiei ſuæ trienti ducto in lineam perpendicularem è centro ſuo in baſem.

14. Si linea ς ſecetur ſecundum extremam & mediam rationem, ut σ ſit majus ſegmentum, & τ minus: Dico $\sigma q = \varsigma\tau = \sigma\tau + \tau q$. per 11 & 3 e 2.

15. $Q{:}\,\frac{1}{2}\varsigma + \sigma{:} = 5Q{:}\,\frac{1}{2}\varsigma$. Nempe $\frac{1}{4}\varsigma q + \varsigma\sigma + (\sigma q)\, \varsigma\tau$. Eſt 1 & 2 e 13.

16. $Q{:}\,\frac{1}{2}\sigma + \tau{:} = 5Q{:}\,\frac{1}{2}\sigma$. Nempe $\frac{1}{4}\sigma q + (\sigma\tau + \tau q)\, \varsigma\tau$. Eſt 3 e 13.

Quare

Quare $\sigma.\tau::\tau.\sigma-\tau$.

17. $\varsigma q+\tau q=3\sigma q$. Nempe $\sigma q+(2\sigma\tau+\tau q+\tau q)2\varsigma\tau$. Eſt 4 e 13.

18. $\varsigma+\tau.\varsigma::\varsigma.\sigma$. Nempe $\varsigma+\sigma.\varsigma::\sigma+\tau.\sigma$. Eſt 5 e 13.

19. Si ς ſit ir, σ erit *Apotome*. Nam quia per 15, $\frac{1}{2}\varsigma+\sigma.\frac{1}{2}\varsigma::\sqrt{q5}.1$: Erunt $\sigma+\frac{1}{2}\varsigma, \frac{1}{2}\varsigma$ $ir\ \beth$, per def: 6 e 10. Et per 37 e 10, erit $\sigma+\frac{1}{2}\varsigma+\frac{1}{2}\varsigma$ *Binomium*. Ergo per 74 e 10, $\sigma+\frac{1}{2}\varsigma-\frac{1}{2}\varsigma$ *Apotome*, hoc eſt σ.

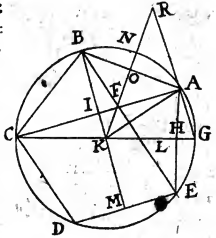
Item ſi ς ſit ir, τ erit *Apotome*. Nam per 61 & 98 e 10, $\frac{\sigma q}{\varsigma\, ir}$ (hoc eſt τ) *Apotome*. vide 14. Eſt 6 e 13.

20. Si ς ſit ſubtendens angulum pentagoni ordinati; erit σ latus pentagoni. Dico in Schemate, AC. CF::CF.AF : Et CF=CB=AB. Nam quia trianguli BCF, omnes tres ang = $\frac{10}{5}$ recti : è quibus ang: BCF = $\frac{2}{5}$ recti; & ang: CBF = $\frac{4}{5}$ recti: tertius igitur ang: CFB = $\frac{4}{5}$ recti: quare CF = CB = AB. Et quia liquet tri: ACB, BAF ſim: Erit AC.AB :: AB. AF: Ergo. Eſt 8 e 13.

Confect. Et ſi ex angulo B per centrum, ad oppoſitum latus pentagoni, ducatur BKM, ſecans ipſam AC in I : ſecabitur etiam linea BM ſecundum extremam & mediam rationem in puncto I. Nam quia in tri: EBM, lateri EM parallela eſt FI : erit per 2 e 6, IM. IB :: FE. BF :: CF. FA. Ergo.

21. Si circuli alicujus radius ſit σ, erit τ latus decagoni. Nam quia arcus ABC=2GAN, erit ang: RKG=KGA=KAG : ideoque tri: RGK, KAG ſim.

ſim. Eſtque RG. KG:: KG.AG. Atque AR = KG, quia ang: ½ RKG = KRG. Secatur igitur RG ſecundum mediam & extremam rationem in puncto A. Ergo latus decagoni AG eſt minus ſegmentum. Eſt 9 e 13. Quare etiam ſi ſ ſit Radius, erit σ latus decagoni.

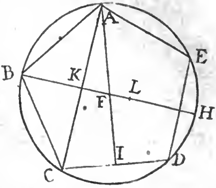


22. Perpendicularis KH vel KO, a centro in latus pentagoni ordinati, æquatur ſemiſummæ Radii & lateris decagoni, Nempe KO = ½RG = ½ KR. Nam quia KR = RG; ſublato utrinque radio, manebit RN = AG. Eſtque KO = RO, per 2 e 3. Eſt 1 e 14.

23. Quadratum lateris pentagoni ordinati, minus quadrato Radii, æquatur quadrato lateris decagoni: Nempe AEq - KGq = AGq. Nam quia AHq + GHq = AGq: Et quia KG ſecatur med: & extr: ratione in L; eſtque KL = AG: Erit AEq + GLq = 4AGq: Et per 17, KGq + GLq = 3AGq. Fiat ſubductio. Eſt 10 e 13.

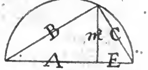
24. Quadratum lateris pentagoni, plus quadrato lineæ ſubtendentis angulum pentagoni, æquatur quinque quadratis Radii. Nempe in ſchemate præcedente, AEq + CAq = 5KGq. Nam CAq + AGq = 4KGq: & per 23, AEq - AGq = KGq. Fiat additio. Eſt hæc 3 e 14.

25. Si circuli Radius ſit rationalis, latus pentagoni inſcripti erit irrationale, Nempe Minor. Nam quia triang: rect. AIC, AKF ſim: erit CI. ½ AC :: KF. ½AF: ideoque 2CI. CK::KF. ¼ AF = FL, qui quadrans eſt Radii: Et CD+CK. CK::KL. FL. At per 2c, ſi CD ſit σ, CK erit ½σ: quare ſi FK ſit σ, FL erit ½σ: & per 15, KLq = 5FLq. Eſt autem BLq = 25FLq: quare BL. KL:: √q25. √q5, √⅁, per def: 6 e 10: Et ſic ipſorum quadrata: unde etiam BLq. BLq-KLq :: 25. (25-5) 20 :: 5. 4: Et BL. √u: BLq-KLq:: √q5. 2, ⊓. Quare BL-KL, nempe BK eſt √ *Apotome* IV, per def: & 47 e 10: quippe oſtenſum eſt, A, E √⅁; A⊓ √X; & A⊓R. Item BCq = BKq+CKq = BKq+BK×BH (per 35 e 3) = √BK× √BH. Ergo per 95 e 10, BC eſt √ *Apot.* IV, hoc eſt Minor. Eſt 11 e 13.



26. In triang. rect. cujus Hypotenuſa Z dividitur in ſegmenta A, E, perpendiculari ex angulo recto demiſſo, Erit 1°, ZA = Bq: & ZE = Cq: & AE = mq.

II°, A.E:: Aq.mq:: mq.Eq:: Bq.Cq
III, Z.A:: Zq.Bq:: Bq.Aq:: Cq.mq
IV°, Z.E:: Zq.Cq:: Cq.Eq:: Bq.mq



27. Si triangulum æquilaterum inſcribatur circulo: 1° perpendicularis è centro in latus æquatur $\frac{1}{2}$ Radii. Ideoque altitudo Δ^{i}, ſive perpendicularis è vertice in baſem æquatur $\frac{3}{2}$ Radii. 2°, Q:dia. Q:lat: Δ^{i}::4.3: ideoque Q:Rad. Q:lat:Δ^{i}::1.3. Eſt 12 e 13.

3°, Q:lat. Q:alt: - ::4.3. ſc: 3.$\frac{3}{4}$. Eſt 12 e 14.

4°, Area trianguli æquilateri $\sqrt{\frac{27}{16}}$, æquatur quadrato mediæ proportionalis inter altitudinem & ſemiſſem lateris: vel inter latus & $\frac{1}{2}$ altitud. Eſt 29 e 14.

5°. Q:lat:Δ^{i}. Q:perpend: è cent: in baſ::3.$\frac{1}{4}$ eſt. 18 e 14

28. Si quadratum inſcribatur circulo: latus ipſius erit $\sqrt{2}$: Et Q:lat:□^{i}. Q:dia::1.2.

29. Si eidem circulo inſcribatur tum, triangulum æquilaterum, tum quadratum: 1°, Q: lat:Δ^{i}. Q:lat: □^{i}::3.2: per 27 2°, & 28. Eſt 16 e 14.

2°, Q:alt:Δ^{i}. Q:lat:□^{i}::9.8: per 27 3°, & 29 1°.

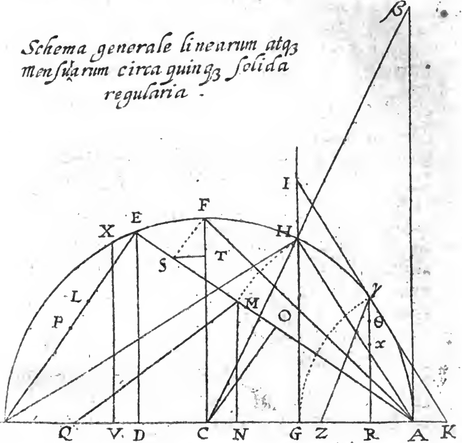
3°, Δ.□::$\sqrt{27}$.8: ſcil: $\sqrt{\frac{27}{16}}$.$\sqrt{4}$.

30. Latera quinque ſolidorum regularium exponere, & inter ſe comparare. Eſt 13, 14, 15, 16, 17, 18, e 13. Eſto AB vel ipſi perpendicularis Aβ, axis ſphæræ, & C centrum; ducatur Cβ ſecans circulum ſphæræ in H; ducaturque HG parallela ipſi Aβ: eritque GH=2CG; & per 47 e 1, Rq=5Q:$\frac{1}{2}$HG; & per 15, ſi HG ſit 5, AG eſt σ; & per 21, ſi HG ſit Radius, erit AG latus decagoni; & per 23, AH latus pentagoni.

Menſuretur CV=CG; & VX=GH. Et è centro erigatur CF; jungaturque AF. tum dividatur axis AB trifariam, ſic ut BD ſit $\frac{1}{3}$, & AD $\frac{2}{3}$: ducanturque per-

perpendicularis DE, & chordæ AE, BE. Secetur BE med: et extr: ratione in puncto L.

Statuaturque GI=BE; & IK ipſi AH parallela: Et ſic erit GK=BL ſegmentum majus.



His diligenter memoriæ mandatis, ad ipſa quinque corpora regularia declaranda pergemus.

De Tetraëdro. Latus (4) eſt AE; & DE Radius circuli ambientis baſem ipſius triangulam. Nam per 26 IV°, AEq. DEq::AB.BD::3.1::Q:lat: Δi. Q: Rad:

Rad: per 27 2°. Et CD & perpendicularis è centro ſphæræ in baſem, ſcil: $\frac{1}{6}$ axis. Et $\frac{1}{2}$ DE eſt perpendicularis è centro baſis in latus, per 27 1°.

De Hexaëdro. Latus (6) Eſt BE vel GI. Nam per 26 IV, ABq.BEq::AB.BD::3.1. Et quia per 26 1°, AEq=2BEq: erit ABq=3BEq (hoc eſt quadratum diagonii Cubi æquatur tribus quadratis lateris): Eſtque per 28, Q: lat:□ⁱ.Q:dia circ::1.2::BEq.AEq. Quarè $\frac{1}{2}$ AE eſt Radius circuli ambientis baſem triangulam (6). Liquet etiam quod $\frac{1}{2}$BE æquatur perpendiculari, tum è centro ſphæræ in baſem, tum è centro baſis in latus. Denique quia ABq.BEq::6.2::Q:axis. Q:lat:(6): Erit 2Q: axis=6Q: lat (6); quæ ſuperficies eſt Cubica.

De Octaëdro. Latus (8) eſt AF vel AS. Nam (8) conſtat duabus pyramidibus quadrangulis, quarum altitudo eſt ſemiaxis: & Q axis Q:lat (8)::2.1. & quia per 27 2°, Q:lat Δⁱ, quod eſt Q:lat (8). Q:diam: circuli ambientis::3.4: Erit Q: axis. Q: diam::3.2. Ductaq; ST parallela axi, quia ACq.CTq::ABq.BEq:: 3.1: Eſtque ASq=AFq=$\frac{1}{2}$ABq: quare CTq=$\frac{1}{3}$BEq=$\frac{1}{2}$AEq: ideoque CT=$\frac{1}{2}$AE; qui Radius eſt circuli ambientis tum baſem (6), tum baſem (8). Et ſi AS vel AF ſit latus Δⁱ, erit CT Radius circuli ambientis per 27 2°. Et $\frac{1}{2}$CT perpendicularis è centro Δⁱ in latus, per 27 1°. Eſt autem ſuperficies (6)=12 BE×$\frac{1}{2}$BE; & ſuperficies (8)=12AF×$\frac{1}{2}$CT, quod ſatis liquet: Quare BE×$\frac{1}{2}$BE. AF×$\frac{1}{2}$CT::ſuperf: (6).ſuperf: (8):: (6).(8)::BE.AC. Eſt 27 e 14. Quoniam AFq.ACq:: BEq.CTq=$\frac{1}{3}$BEq.

De

De Icoſaëdro. Latus)20) eſt AH vel AM. Nam Radiis GH & VX æqualibus cogitentur duo circuli deſcribi, perpendiculariter inſiſtentes plano AHXB; atque in ipſis includi, tum pentagonum ordinatum lateris AH, tum decagonum lateris AG : ſic ut punctum H ſit angulus pentagoni, & X decagoni : unde anguli pentagoni in uno circulo perpendiculariter imminebunt angulis decagoni in altero, ad diſtantiam GV = GH. Et è ſingulis angulis unius pentagoni ducantur duæ hypotenuſæ ad angulos alterius utrinque proximos : Item ex ſingulis angulis utriuſque pentagoni, in proximum axis terminum A vel B, ducantur hypotenuſæ : Quæ quidem omnes, hypotenuſæ erunt triangulorum rectangulorum, quorum Cathetus æqualis eſt Radio GH, & baſis lateri decagoni AG; ideoque ſingulæ æquales lateri pentagoni AH. Quare deſcripta erit figura conſtans 20 triangulis æquilateris & æqualibus. Includi autem angulos illos omnes ſphæra, patet ex angulo H : nam circumvolutus ſemicirculus AHXB reliquos ſimiliter angulos perſtringet. Eſt igitur GH Radius circuli ambientis pentagonum (20): & $\sqrt{}$ quia 5.1::CHq.GHq: eſt autem CH = $\frac{1}{2}$AB, & CG = $\frac{1}{2}$GH : Atque id circo AH latus (20) eſt $\sqrt{}$, nempe Minor, per 25. Tum demiſſa MN perpendiculari in axem, ſtatuatur MQ = AC, $\frac{1}{2}$ axis : erit MN Radius circuli circa baſem, per 27 2; quia AM.MN::AE.DE::3.1: Et $\frac{1}{2}$ MN perpendicularis è centro baſis in latus, per 27 1°: Et NQ perpendicularis è centro ſphæræ in baſem; quia ibi Q

Rad: per 27 2°. Et CD & perpendicularis è centro ſphæræ in baſem, ſcil: $\frac{1}{6}$ axis. Et $\frac{1}{2}$ DE eſt perpendicularis è centro baſis in latus, per 27 1°.

De Hexaëdro. Latus (6) Eſt BE vel GI. Nam per 26 IV, ABq.BEq::AB.BD::3.1. Et quia per 26 1°, AEq = 2BEq: erit ABq = 3BEq (hoc eſt quadratum diagonii Cubi æquatur tribus quadratis lateris): Eſtque per 28, Q: lat:□ⁱ.Q:dia circ::1.2::BEq.AEq. Quarè $\frac{1}{2}$ AE eſt Radius circuli ambientis baſem triangulam (6). Liquet etiam quod $\frac{1}{2}$BE æquatur perpendiculari, tum è centro ſphæræ in baſem, tum è centro baſis in latus. Denique quia ABq.BEq::6.2::Q:axis. Q:lat:(6): Erit 2Q: axis = 6Q: lat (6); quæ ſuperficies eſt Cubica.

De Octaëdro. Latus (8) eſt AF vel AS. Nam (8) conſtat duabus pyramidibus quadrangulis, quarum altitudo eſt ſemiaxis: & Q axis Q:lat (8)::2.1. & quia per 27 2°, Q:lat Δⁱ, quod eſt Q:lat (8). Q:diam: circuli ambientis::3.4: Erit Q: axis. Q: diam::3.2. Ductaq; ST parallela axi quia ASq.CTq::ABq.BEq:: 3.1: Eſtque ASq = AFq = $\frac{1}{2}$ABq: quare CTq = $\frac{1}{3}$BEq = $\frac{1}{6}$AEq: ideoque CT = $\frac{1}{2}$AE; qui Radius eſt circuli ambientis tum baſem (6), tum baſem (8). Et ſi AS vel AF ſit latus Δⁱ, erit CT Radius circuli ambientis per 27 2°. Et $\frac{1}{2}$CT perpendicularis è centro Δⁱ in latus, per 27 1°. Eſt autem ſuperficies (6) = 12 BE×$\frac{1}{2}$BE; & ſuperficies (8) = 12AF×$\frac{1}{2}$CT, quod ſatis liquet: Quare BE×$\frac{1}{2}$BE. AF×$\frac{1}{2}$CT::ſuperf: (6).ſuperf: (8):: (6).(8)::BE.AC. Eſt 27 e 14. Quoniam AFq.ACq:: BEq.CTq = $\frac{1}{3}$BEq. De

De Icoſaëdro. Latus)20) eſt AH vel AM. Nam Radiis GH & VX æqualibus cogitentur duo circuli deſcribi, perpendiculariter inſiſtentes plano AHXB; atque in ipſis includi, tum pentagonum ordinatum lateris AH, tum decagonum lateris AG: ſic ut punctum H ſit angulus pentagoni, & X decagoni: unde anguli pentagoni in uno circulo perpendiculariter imminebunt angulis decagoni in altero, ad diſtantiam GV = GH. Et è ſingulis angulis unius pentagoni ducantur duæ hypotenuſæ ad angulos alterius utrinque proximos: Item ex ſingulis angulis utriuſque pentagoni, in proximum axis terminum A vel B, ducantur hypotenuſæ: Quæ quidem omnes, hypotenuſæ erunt triangulorum rectangulorum, quorum Cathetus æqualis eſt Radio GH, & baſis lateri decagoni AG; ideoque ſingulæ æquales lateri pentagoni AH. Quare deſcripta erit figura conſtans 20 triangulis æquilateris & æqualibus. Includi autem angulos illos omnes ſphæra, patet ex angulo H: nam circumvolutus ſemicirculus AHXB reliquos ſimiliter angulos perſtringet. Eſt igitur GH Radius circuli ambientis pentagonum (20): & √r quia 5.1::CHq.GHq: eſt autem CH = $\frac{1}{2}$AB, & CG = $\frac{1}{2}$GH: Atque idcirco AH latus (20) eſt √√r, nempe Minor, per 25. Tum demiſſa MN perpendiculari in axem, ſtatuatur MQ = AC, $\frac{1}{2}$ axis: erit MN Radius circuli circa baſem, per 27 2°; quia AM.MN::AE.DE::3.1: Et $\frac{1}{2}$ MN perpendicularis è centro baſis in latus, per 27 1°: Et NQ perpendicularis è centro ſphæræ in baſem; quia ibi Q

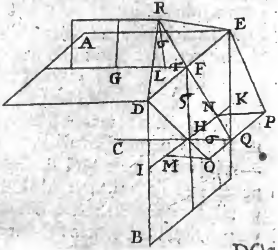
 eſt

eſt inſtar centri ſphæræ. Denique BEq. GHq::5.3 : Nam BEq. ABq::1.3: & ABq. GHq::CHq. CGq:: 5.1.

De Dodecaëdro. Latus (12) eſt BL vel GK, in præcedente ſchemate : & BE vel GI (latus (6)) ſubtendit angulum baſis pentagonæ (12). Nam in ſequente ſchemate, deſcribantur duæ baſes (6), AD EB, quarum commune latus eſt DE; & centrum ſphæræ C; & centrum baſis unius G, alterius H. A centro G ducatur GF perpendicularis lateri DE; & per centrum H ducatur IHK ipſi DE parallela. Erunt igitur GF,HI,HK, ſemiſſes lateris (6): ſecentur ſingulæ in στ punctis L,M,N; ut majus ſegmentum ſit ubique centro proximum : & in punctis L,M,N, erigantur tres perpendiculares LR,MO,NP, æquales ipſi majori ſegmento: & ducatur OP, latus(12): eſt enim IK.OP::ς.σ::BE.BL, ſchematis præcedentis. Ducantur etiam DO,DR,EP,ER, quæ cum OP includunt pentagonum, baſem quidem(12). Nam

1°. Pentagonum DOPER eſt in uno plano : Eſt enim RFQ una recta linea, per 32 e 6.

2°, Eſt æquilaterum : eſt enim



DOq

DOq = MOq pl. DIq+IMq, hoc eſt, 3MOq, per 17. At etiam 4MOq = OPq. Et ſic de cæteris.

3°, Eſt æquiangulum: Eſt enim DPq = DIq pl. INq+NPq, hoc eſt, 3DIq, per 18 & 17. At etiam 4DIq = DEq. Et ſic de cæteris.

4°, Circumſcribitur ſphæra: Eſt enim CPq = CQq+QPq, hoc eſt, 3CHq, per 18 & 17. at Q: axis. Q: lat(6) :: 3.1 :: Q: $\frac{1}{2}$axis. Q: $\frac{1}{2}$ lat(6). Et ſic de reliquis.

5°, Circa (6) deſcribentur 12 ejuſmodi pentagona: Cum enim per II, ſint in (6) latera 12; unicuique lateri ſuum adhærebit pentagonum; ſicut intuenti perſpicuum erit.

6°, Latus (12) eſt Apotome: Eſt enim DE latus (6) ϰ⊐ axi: at per 19, ſi ς (DE) ſit ϰ, σ (OP) erit Apotome.

His ſic oſtenſis, ad prius illud ſchema redeundum denuò erit: In quo menſuretur Kγ = KG = BI. lateri (12): & demittatur γR. Erit per 23, γR Radius circuli circa baſem pentagonam: Et per 22, Rθ, ſcil: $\frac{1}{2}$Rγ+$\frac{1}{2}$RK, eſt perpendicularis è centro baſis in latus. Eſt autem Rγ = MN: Nam quia (3BEq) 3GIq = Q: axis = 5GHq: erit 5.3 :: GIq. GHq :: GKq. GAq :: GIq+GKQ. GHq+GAq: hoc eſt, per 20 & 24: 5Rγq. AMq = 3MNq, per 26 IV°. Quare 3×5Rγq = 5×3MNq. Eſtque QN perpendicularis à centro ſphæræ in baſem. Eſtque ſuperficies (20) = 30 AH×$\frac{1}{2}$MN: & ſuperficies (12) = 30GK×Rθ, quod ſatis conſtat. Quare AH×$\frac{1}{2}$MN. GK×Rθ :: ſuperf (20). ſuperf (12) :: (20).(12).

26. Si axis ſphæræ æqualis ſit, tum $\sqrt{u: \varsigma q + \tau q}$ unius lineæ, tum $\sqrt{u: \varsigma q + \tau q}$ alterius lineæ: erit σ latus (20); & τ latus (12). Nam in Schema priore generali, $\varsigma . \sigma$ :: GH.AG::BH.AH: At ABq = BHq + AHq. Item ABq = 3BEq = Q:BE+BL; pl BLq: hoc eſt, $3\varsigma q$ = Q:$\varsigma + \tau$:pl σq. Eſt enim $\sigma q = \varsigma\tau$. Eſt 23 e 14.

27. $\sqrt{u: \varsigma q + \sigma q}$. $\sqrt{u: \varsigma q + \tau q}$::lat (6). lat (20) hoc eſt, Kγ. Zγ :: BE.AH, vel GI.AM: ſecta ſcil: KZ = Rγ med: & extr: ratione in puncto R. Nam per 26 IV°, AMq = 3Rγq: Et per 17, Zγq = 3RKq. Quare AM. Zγ::Rγ. RK::$\varsigma . \sigma$::GI. KG. Eſt 10 e 14.

38. Latus (6). Latus (20)::ſuperf (12). ſuperf (20). hoc eſt GI. AM:: KG×Rγ. AM×$\frac{1}{2}$Rγ. Nam KG×Rγ = GI×$\frac{1}{3}$Rγ. Eſt enim GI.KG::$\varsigma . \sigma$::Rγ + RK. Rγ::($\frac{1}{2}$Rγ + $\frac{1}{2}$RK) Rγ. $\frac{1}{3}$Rγ, per 18. Eſt 9 e 14.

39. Q:perpendic: è centro ſphæræ in baſem (4). Q:perpend: è centro ſphæræ in baſem (8) :: 1.3:: CDq.$\frac{1}{4}$BEq.

40. Q: lat (4). Q: lat (8)::Baſis (4). Baſis (8). Nam AEq.ABq::2.3: Et ABq.AFq::4.2. Eſt 14 e 14. Hinc conſectarium eſt,

Quo:l, Superf 4).ſuperf (8)::2.3:ſcil:4×4.8×3.

41. Q: (4). Q: (8) :: 4.27. per 39 & conſect: 40: Nempe × $\left\{ \begin{matrix} 1.3 \\ 2.9 \end{matrix} \right\}$. Eſt 17 e 14.

42. Baſis (6). Baſis (8)::8.$\sqrt{27}$: Nempe $\frac{4}{3}$.$\sqrt{\frac{3}{4}}$.

43. Baſis (4). Baſis (6)::$\sqrt{3}$.2::altit: Δ^{i}(4).latus Δ^{i}(4): nempe $\frac{2}{3}$BE. AE. Eſt 30 e 14.

44. Superf (4). Superf (6) ::1.$\sqrt{3}$: Nempe

$\sqrt{\frac{4}{3}}$

$\sqrt{\frac{4}{3}} \times 4.8$: hoc eſt, $\Delta^m (4) \times 4.2Q$: axis.

45. Tria $(4) = (6)$: per 44 & 39: Nempe $\times \left\{ \begin{matrix} 1.\sqrt{3} \\ 1.\sqrt{3} \end{matrix} \right\}$ eſt 32 e 14. Hinc conſectarium eſt,

Quod $\left\{ \begin{matrix} \text{Priſma baſis \& altitudinis } (4) = (6). \text{ Et} \\ \text{Pyramis baſis \& altitudinis } (6) = (4). \end{matrix} \right.$

46. $(8).3(4)$ :: latus (8). latus (4): Nempe 2. $(\sqrt{\frac{16}{27}} \times 3)\sqrt{\frac{16}{3}} :: \sqrt{2}.\sqrt{\frac{8}{3}}$. Eſt 22 e 14.

47. Si latus $(8) = \sqrt{u: \sigma q + \tau q}$, erit latus $(20) = \sqrt{2\tau q}$. Nam BH+HA ſecatur med.: & extr: ratione in H: Eſtque $2\tau q + 2\varsigma q = 2AFq = ABq = BHq + AHq$. Ergo $AHq = 2\tau q$. Eſt 24 e 14.

48. Si latus $(8) = \sqrt{u: \frac{1}{2}\varsigma q + \frac{1}{2}\tau q}$, Erit latus $(12) = \tau$. Nam GI+GH ſecatur med: & extre: ratione in G: Eſtque $\varsigma q + \tau q = 2AFq = ABq = 3GIq = Q: GI + GK: + GKq$. Ergo $GKq = \tau q$. Eſt hæc 25 e 14.

49. Si latus $(4) = \sqrt{u: \sigma q + \tau q}$, erit latus $(20) = \sqrt{\frac{2}{3}\tau q}$. Nam BH+HA ſecatur media & extr: ratione in H: & $\frac{2}{3}\sigma q + \frac{2}{3}\tau q = \frac{2}{3}AEq = ABq = BHq + AHq$. Ergo $AHq = \frac{2}{3}\tau q$. Eſt hæc 26 e 14.

50. Si latus $(6) = \sqrt{u: \sigma q + \tau q}$, erit latus $(20) = \sqrt{3\tau q}$. Nam BH+AH ſecatur med: & extr: ratione in H: & $3\sigma q + 3\tau q = 3GIq = ABq = BHq + AHq$. Ergo $AHq = 3\tau q$

51. Si latus $(6) = \sqrt{u: \varsigma q + \tau q}$, erit latus $(12) = \sqrt{3\tau q}$. Nam GI+GK ſecatur med: & extrem: ratione in H: & $3\varsigma q + \sigma q = 3GIq = Q: GI + GK: + GKq$. Ergo $GKq = 3\tau q$.

52. Si axis ſphæræ ſit $\kappa\tau$, ſuperficies tum (4), tum

tum (8); erit mr. Nam quia 3.2::Q:axis. A Eq:erit Q:lat:(4)$=\frac{1}{3}$Q:axis:eſt etiam Q:lat:(8)$=_{2}$Q:axis: ſcil: utrumque ĸr: quare & ipſorum latera ſunt ĸr. At in Δ°, per 27,3, Latus. altitud::2.√3, ĸr ⊐. ergo per 22 e 10, area Δ^{i} eſt mr. Eſt 13 e 14.

Notandum autem, quod in his quæ tum de elemento X, tum de V corporibus regular: ſcripta ſunt, propoſitionum numerus eſt juxta Ch: Clau.

Corporum

Corporum quinque regularium menſuræ, ad axem ſphæræ 2. Conſulatur Schema generale.

I. *In Tetraëdro.*

AE latus (4), eſt $\sqrt{\frac{8}{3}}$: 1,632932.

DE ſemidiameter circuli ambientis baſem triangulam (4), eſt $\sqrt{\frac{8}{9}}$: 0,942809.

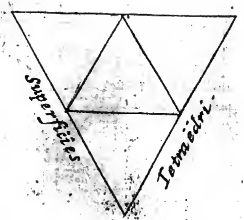
Altitudo baſis, eſt 1,414213.

Area baſis (4), eſt 1,154657.

Superficies (4), eſt 4,618628.

CD perpendicularis è centro ſphæræ in baſem (4), eſt $\frac{1}{3}$, 0,333333.

Soliditas (4), eſt 0,513216.



II. *In Hexaëdro.*

BE latus (6), eſt $\sqrt{\frac{4}{3}}$; 1,154700.

CT eſt ſemidiameter circuli ambientis baſem quadrangulam (6), $\sqrt{\frac{2}{3}}$: 0,816490.

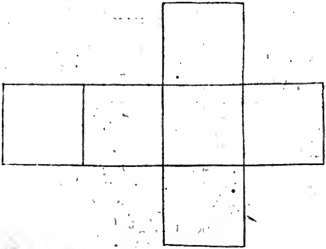
Area baſis (6), eſt $\frac{4}{3}$: 1,333333.

Superficies (6), eſt 8: Nempe bina quadrata axis, ſphæræ.

$\frac{1}{2}$BE perpendicularis è centro ſphæræ in baſem (6) eſt $\sqrt{\frac{1}{3}}$: 0,577175.

Soliditas (6), eſt 1,539600.

Superficies Hexaëdri.



III. *In Octaëdro.*

AF latus (8), eſt $\sqrt{2}$: 1,414213.

CT eſt ſemidiameter circuli ambientis baſem triangulam (8), eſt $\sqrt{\frac{2}{3}}$: 0,816490.

Altitudo baſis (8), eſt 1,224735.

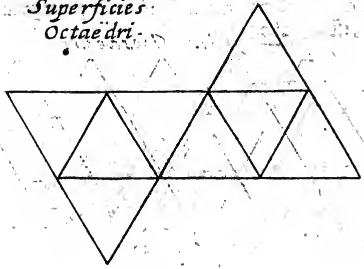
Area baſis (8), eſt 0,866018.

Superficies (8), eſt 6,928144.

$\frac{1}{3}$BE perpendicularis è centro ſphæræ in baſem (8), eſt $\sqrt{\frac{1}{3}}$: 0,577175.

Soliditas (8), eſt 1,333333.

Superficies Octaedri.



IV. *In Icoſaëdro.*

AH latus (20), eſt $\sqrt{u}: 2-\sqrt{\frac{4}{5}}: 1,105573$.

MN=Rγ ſemidiameter circuli ambientis baſem triangulam (20), eſt $\sqrt{u}: \frac{2}{3}-\sqrt{\frac{4}{45}}: 0,607062$.

Altitudo baſis (20), eſt 0,910593.

Area baſis (20), eſt 0,503362.

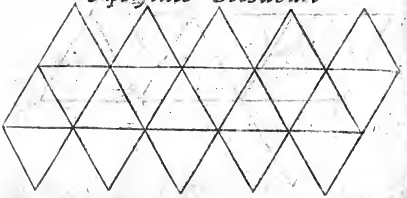
Superficies (20), eſt 10,067240.

QN perpendicularis è centro ſphæræ in baſem (20). eſt $\sqrt{u}: \frac{1}{3}+\sqrt{\frac{4}{45}}: 0,794654$.

Soliditas (20), eſt 2,666658.

GH eſt ſemidiameter circuli ambientis pentagonum (20), eſt $\sqrt{\frac{4}{5}}: 0,894427$.

Superficies Icoſaëdri



V.

V. *In Dodecaëdro.*

GK = BL latus (12), eſt $\sqrt{\frac{5}{3}} - \sqrt{\frac{1}{3}}$: 0,713642.

$R\gamma$ = MN ſemidiameter circuli ambientis baſem quinquangulam (12), eſt $\sqrt{u: \frac{2}{3} - \sqrt{\frac{4}{45}}}$: 0,607062.

$R\theta = \frac{1}{2}R\gamma + \frac{1}{2}RK$, perpendicularis è centro baſis in latus, eſt 0,49112.

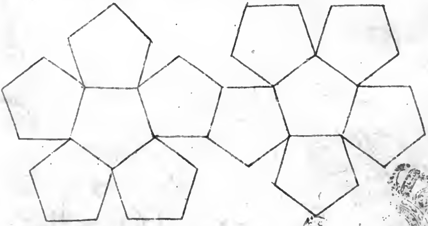
Area baſis (12), eſt 0,876211.

Superficies (12), eſt 10,514532.

QN perpendicularis è centro ſphæræ in baſem (12), eſt $\sqrt{u: \frac{1}{3} + \sqrt{\frac{4}{45}}}$: 0,794654.

Soliditas (12) eſt 2,785137.

Superficies Dodecaëdri.



FINIS.

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